

Analytical Skills

EPEA204

Edited by:
Harveer Minhas



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Analytical Skills

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Unit 01: Number System

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Summary

Keywords

Self Assessment

Answers for Self Assessment

Review Questions

Further Readings

Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Different types of numbers
- Multiplication and squaring short cuts.
- Get Distributive Laws.
- Divisibility Rules.
- Tests of Divisibility
- Methods of Finding L.C.M.
- Methods of Finding H.C.F.

Introduction

Numbers are an integral part of our everyday lives, right from the number of hours we sleep at night to the number of rounds we run around the racing track and much more. In math, numbers can be even and odd numbers, prime and composite numbers, decimals, fractions, rational and irrational numbers, natural numbers, integers, real numbers, rational numbers, irrational numbers, and whole numbers. In this chapter, we'll get an introduction to the different types of numbers and to all the concepts related to it. Numbers form the basis of mathematics. We should befriend numbers in order to understand math. Numbers are of various kinds. We have a long list that includes ordinal numbers, consecutive numbers, odd numbers, even numbers, natural numbers, whole numbers, integers, real numbers, rational numbers, irrational numbers, and complex numbers.

Along with numbers, we come across the interesting world of factors and multiples. This world includes prime numbers, composite numbers, co-prime numbers, perfect numbers (yes, numbers could be perfect!) HCF, LCM, and prime factorization

1.1 Types of Numbers

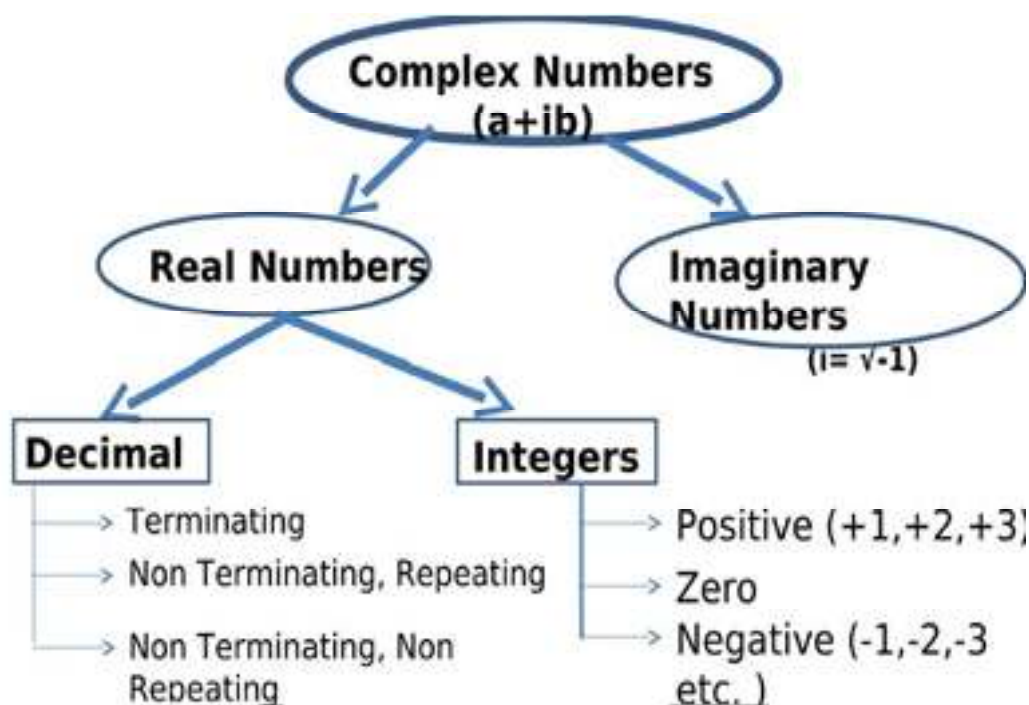


Fig. 1.1 Classification of Number System

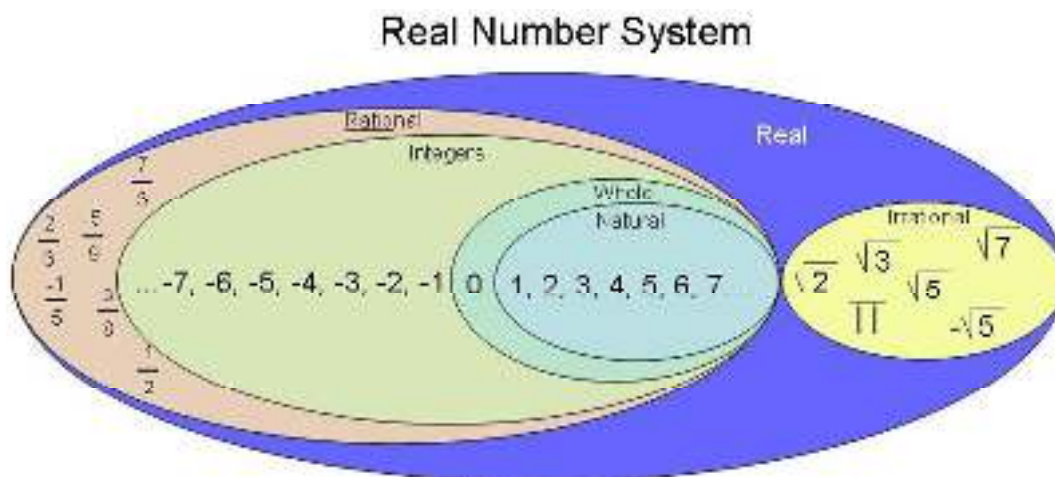


Fig. 1.1.1 Graphical Representations of Real Number System

1. Natural Numbers: Counting numbers 1, 2, 3, 4, 5, are called natural numbers.
2. Whole Numbers: All counting numbers together with zero form the set of whole numbers.
Thus,
 - (i) 0 is the only whole number which is not a natural number.
 - (ii) Every natural number is a whole number.
3. Integers: All natural numbers, 0 and negatives of counting numbers i.e., {..., -3, -2, -1, 0, 1, 2, 3,} together form the set of integers.

(i) Positive Integers: $\{1, 2, 3, 4, \dots\}$ is the set of all positive integers.

(ii) Negative Integers: $\{-1, -2, -3, \dots\}$ is the set of all negative integers.

(iii) Non-Positive and Non-Negative Integers: 0 is neither positive nor negative. So, $\{0, 1, 2, 3, \dots\}$ represents the set of non-negative integers, while $\{0, -1, -2, -3, \dots\}$ represents the set of non-positive integers.

4. Even Numbers: A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers: A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. Prime Numbers: A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers up to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

How to find if a number is prime or not?

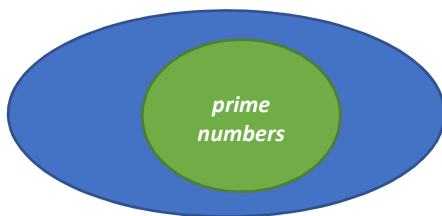
N is a prime number if it is not divisible by numbers lesser than \sqrt{N} .



For Example: 191 is a prime number since it is not divisible by 2, 3, 5, 7, 11 and 13 [numbers less than $\sqrt{191} (\approx 14)$].

Note: Prime numbers will always be in the form $(6k \pm 1)$ where $k = 1, 2, 3, \dots$

But not all $(6k \pm 1)$ will be a prime number.



$$6k \pm 1$$

7. Composite Numbers: Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.

Note: (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 and 100.

8. Co-primes: Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g.,

(2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

8. Rational Numbers: The numbers of the form $\frac{p}{q}$

where p and q are integers

and $q \neq 0$, are known as rational numbers,

The set of all rational numbers is denoted by Q.

That is, $Q = \{x: x = \frac{p}{q}; p, q \in I, q \neq 0\}$

Since every natural number 'a' can be written as $\frac{a}{1}$, Therefore, every natural number is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$. Therefore, every integer is a rational

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or in non-terminating repeating decimals.



For example

$$\frac{1}{5} = 0.2, \quad \frac{1}{3} = 0.3333 \dots, \quad \frac{22}{7} = 3.1428571428 \dots, \quad \frac{8}{44} = 0.18181818 \dots, \text{ etc.}$$

The recurring decimals have been given a short notation as

$$0.333\dots = 0.\bar{3}$$

$$4.1555\dots = 4.1\bar{5}$$

$$0.323232\dots = 0.\overline{32}$$

9. Irrational numbers: Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g., $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π , etc.

Note that the exact value of π is not $\frac{22}{7}$. $\frac{22}{7}$ is rational while π is irrational number. $\frac{22}{7}$ is approximate value of π . Similarly, 3.14 is not an exact value of it.

10. Real numbers: The rational and irrational numbers combined together to form real numbers, e.g., $\frac{13}{21}$, $\frac{2}{5}$, $\frac{-3}{7}$, $\sqrt{3}$, $4 + \sqrt{2}$ etc. are real numbers. The set of all real numbers is denoted by R

Note that the sum, difference or product of a rational and irrational number is irrational, e.g., $3 + \sqrt{2}$, $4 - \sqrt{3}$, $\frac{2}{5} - \sqrt{5}$, $\sqrt[4]{3}$, $-\sqrt[3]{5}$ are all irrational.

11. Even numbers: All those numbers which are exactly divisible by 2 are called even numbers, e.g., 2, 6, 8, 10, etc., are even numbers.

12. Odd numbers: All those numbers which are not exactly divisible by 2 are called odd numbers, e.g., 1, 3, 5, 7, etc., are odd numbers.

13. Composite numbers: Natural numbers greater than 1 which are not prime, are known as composite numbers. For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g., 16 and 17 are relatively prime, although 16 is not a prime number.

1.1.1 Conversion of a decimal number to fraction:



For Example : 6.424242.....

$$\text{Let } x = 6.424242\dots$$

$$100x = 642.424242\dots$$

$$(100x - x) = (642.424242\dots - 6.424242\dots)$$

$$99x = 636$$

$$x = 636/99$$

$$\therefore 6.424242\dots = 636/99$$

1.2 Divisibility Rules

1. Divisibility by 2: - A number is divisible by 2 if the unit's digit is zero or even numbers. For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. Divisibility by 3: - A number is divisible by 3 if the sum of digits in the number is divisible by 3. For example, the number 3792 is divisible by 3 since $3 + 7 + 9 + 2 = 21$, which is divisible by 3.

3. Divisibility by 4: - A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero. For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

4. Divisibility by 5: - A number is divisible by 5 if the unit's digit in the number is 0 or 5. For example,

13520, 7805, 640, 745, etc., are all divisible by 5.

5. Divisibility by 6: - A number is divisible by 6 if the number is even and sum of its digits is divisible by 3. For example, the number 4518 is divisible by 6 since it is even and sum of its digits $4 + 5 + 1 + 8 = 18$ is divisible by 3.

6. Divisibility by 7: - The unit digit of the given number is doubled and then it is subtracted from the

number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7. For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16. Then, $44 - 16 = 28$.

Since 28 is divisible by 7, 448 is divisible by 7.

7. Divisibility by 8: - A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e., 784 is

divisible by 8.

8. Divisibility by 9: - A number is divisible by 9 if the sum of its digits is divisible by 9. For example, the number 19044 is divisible by 9 as the sum of its digits $1 + 9 + 0 + 4 + 4 = 18$ is divisible by 9.

9. Divisibility by 10: - A number is divisible by 10, if it ends in zero.

For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. Divisibility by 11: - A number is divisible by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

For example, in the number 38797, the sum of the digits at odd places is $3 + 7 + 7 = 17$ and the sum of

the digits at even places are $8 + 9 = 17$. The difference is $17 - 17 = 0$, so the number is divisible by 11.

11. Divisibility by 12: - A number is divisible by 12 if it is divisible by 3 and 4.

12. Divisibility by 18: - An even number satisfying the divisibility test of 9 is divisible by 18.

13. Divisibility by 25: - A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero. For example, the number 13675 is divisible by 25 as the

number formed by the last two digits is 75 which is divisible by 25

14. Divisibility by 88: - A number is divisible by 88 if it is divisible by 11 and 8.

15. Divisibility by 125: - A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero. For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

1.3 Multiplicity of Numbers

1. Multiplication by Distributive Law:

(i) $a \times (b + c) = a \times b + a \times c$ (ii) $a \times (b - c) = a \times b - a \times c$.



Example 1. (i) $567958 \times 99999 = 567958 \times (100000 - 1)$

$= 567958 \times 100000 - 567958 \times 1 = (56795800000 - 567958) = 56795232042$. (ii) $978 \times 184 + 978 \times 816 =$

$978 \times (184 + 816) = 978 \times 1000 = 978000$.

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n .

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Ex. $975436 \times 625 = 975436 \times 5^4 = \frac{9754360000}{2^4} = 609647500$.

Multiplication of a given number by 9, 99, 999, etc., that is by $10^n - 1$ Method: Put as many zeros to the

right of the multiplicand as there are nines in the multiplier and from the result subtract the multiplicand and get the answer.



Example 2: Multiply:

(a) 3893 by 99

$$3893 \times 99 = 389300 - 3893 = 385407$$

(b) 4327 by 999

$$4327 \times 999 = 4327000 - 4327 = 4322673$$

(c) 5863 by 99

$$5863 \times 9999 = 58630000 - 5863 = 58624137$$

Multiplication of a given number by 11, 101, 1001, etc., that is, by $10^n + 1$.

Method: Place n zeros to the right of the multiplicand and then add the multiplicand to the number so obtained.



Example 3: Multiply

$$4782 \times 11$$

$$\text{Solution: (a) } 4782 \times 11 = 47820 + 4782 = 52602$$

$$\text{b) } 9836 \times 101$$

$$\text{Solution: - } 9836 \times 101 = 983600 + 9836 = 993436$$

$$\text{c) } 6538 \times 1001$$

$$\text{Solution: - } 6538 \times 1001 = 6538000 + 6538 = 6544538$$

Multiplication of a given number by 15, 25, 35, etc. Method: Double the multiplier and then multiply

the multiplicand by this new number and finally divide the product by 2



Example 4: Multiply

$$7054 \times 15 = \frac{1}{2}(7054 \times 30) = \frac{1}{2}(7054 \times 30) = \frac{1}{2}(211620) = 105810$$

$$\text{b) } 3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850) = 97425$$

$$\text{c) } 4536 \times 35 = \frac{1}{2}(4536 \times 70) = \frac{1}{2}(319410) = 159705$$



Example 5: Multiply

$$3982 \times 5 = \frac{1}{2}(39820) = 19910$$

$$\text{b) } 4739 \times 25 = \frac{1}{2^2}(473900) = 118475$$

$$\text{c) } 7894 \times 125 = \frac{1}{2^3}(7894000) = 986750$$

$$\text{d) } 4863 \times 625 = \frac{1}{2^4}(48630000) = 3039375$$

1.3 Distributive Laws

For any three numbers a, b, c we have

$$a) a \times b + a \times c = a \times (b + c)$$

$$b) A \times b - a \times c = a \times (b - c)$$



Example 6: $438 \times 637 + 438 \times 367 = ?$

$$\text{Solution: } - 438 \times 637 + 438 \times 367 = 438 \times (637 + 367) = 438 \times 1000 = 438000$$



Example 7: $674 \times 832 - 674 \times 632 = ?$

$$\text{Solution: } - : 674 \times 832 - 674 \times 632$$

$$= 674 \times (832 - 632)$$

$$= 674 \times 200$$

$$= 134800$$

Multiplication in Vedic Mathematics

Vedic mathematics is based on quick mental calculation. It manifests the coherent and unified structure of mathematics and the methods are complementary, direct and easy.

When neither the multiplicand nor the multiplier is sufficiently near a convenient power of 10 which can serve as a base, we can choose a multiple or a sub-multiple of a suitable base, as our working base.

The operation is performed with the working base and the final calculation is done with the theoretical base.



Example 8:

$$7 \times 9 = 63 \text{ (adding 7 nine times conventional method)}$$

In Vedic mathematics,

$$7 \mid -3 \quad 1. \text{ Here the nearest and convenient base}$$

$$9 \mid -1 \quad \text{is 10 for both multiplicand and}$$

$$\text{----- multiplier.}$$

$$6 \mid 3 \quad 2. \text{ Hence the working base is same as the theoretical base.}$$

STEP 1: The multiplicand 7 is 3 less than 10 so we write it as $7 \mid -3$. Similarly, multiplier 9 is 1 less than 10 hence we write it as $9 \mid -1$.

STEP 2: The product of deficit (or surplus) of the nearest base gives the unit digit of the resultant. i.e. $-3 \times -1 = 3$

STEP 3: The multiplicand 7 is computed with the deficit of the multiplier 9 and the multiplier 9 is computed with the deficit of the multiplicand 7.

$$\begin{array}{r} 7 \quad -3 \\ \times 9 \quad -1 \\ \hline \end{array}$$

=6. This gives the number of **tens**.

$$9 \quad -1 \quad \text{Hence it is } 60+3 \text{ or simply } 63.$$



Example 9:

$$22 \times 21 = 462$$

$$22 \mid +2 \quad 1. \text{ Here our working base is 20, hence}$$

$$21 \mid +1 \quad \text{the multiplicand and the multiplier}$$

----- are expressed as $22 \mid +2$ & $21 \mid +1$.

23 | 2 2. The theoretical base is 10 and is calculated as

22 +2

=23. This gives not the no. of tens but it

21 +1

is the count of 20's since our working base is 20.

Hence there are 23 no. of 20's. In each 20 there are two 10's. Therefore in 23 no. of 20's there are 46 no. of 10's. $23 \times 2 = 46$ tens. Or $46 \times 10 = 460$. Therefore, required answer is $460 + 2(\text{unit digit}) = 462$.



Example 10:

$$17 \times 23 = 391.$$

$$17 \mid -3 \qquad 17 \nmid -3$$

23 | +3

=20. Unit digit= $(-3 * +3)$

23 + 3

$$= -9$$

20 | -9

Working base = 20. $20 \times 20's = 400$ Therefore, required answer is $400 - 9 = 391$.

Criss Cross Multiplication:



Examples 11: 2×2

32 x 21		
Step 1		$\begin{array}{r} 3 \quad 2 \\ \times 2 \quad 1 \\ \hline \end{array} \quad 2 \times 1 = 2$
Step 2		$\begin{array}{r} 3 \quad 2 \\ \times 2 \quad 1 \\ \hline \end{array} \quad (3 \times 1) + (2 \times 2) = 7$
Step 3		$\begin{array}{r} 3 \quad 2 \\ \times 2 \quad 1 \\ \hline \end{array} \quad (3 \times 2) = 6$
<p style="text-align: center;">6/7/2 Ans : 672</p>		




Examples 12: 4 x 4


$$\begin{array}{r} 4125 \\ \times 0511 \\ \hline \end{array}$$

Step(i)  $1 \times 1 = 1$

Step(ii)  $(2 \times 1) + (1 \times 1) = 3$

Step(iii)  $(1 \times 1) + (2 \times 1) + (1 \times 1) = 4$

Step(iv)  $(4 \times 1) + (1 \times 5) + (2 \times 1) + (1 \times 1) = 15$

Step(v)  $(4 \times 1) + (2 \times 5) + (1 \times 5) = 9$

Step(vi)  $(2 \times 5) + (1 \times 5) = 15$

Step(vii)  $4 \times 0 = 0$

$$\begin{array}{r} 0/0/9/5/8/7/5 \\ \hline \text{Ans: } 2107875 \end{array}$$

1.4 Squaring of Numbers

To square any number ending with 5.

Method: $(A5)^2 = A(A + 1)/25$



Example 12:

(a) $(25)^2 = 2(2 + 1)/25 = 6/25 = 625$

(b) $(45)^2 = 4(4 + 1)/25 = 20/25 = 2025$

(c) $(85)^2 = 8(8 + 1)/25 = 72/25 = 7225$

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To square a number in which every digit is one. Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.



Example 13:

(a) $11^2 = 121$

(b) $111^2 = 12321$

(c) $1111^2 = 1234321$

(d) $222^2 = 2^2 (111)^2 = 4(12321) = 49284$

(e) $3333^2 = 3^2 (1111)^2 = 9(1234321) = 11108889$

To square a number which is nearer to $10x$. Method: Use the formula: $x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$



Example 14:

(a) $97^2 = (97 + 3)(97 - 3) + 32 = 9400 + 9 = 9409$

(b) $102^2 = (102 - 2)(102 + 2) + 22 = 10400 + 4 = 10404$

(c) $994^2 = (994 + 6)(994 - 6) + 62 = 988000 + 36 = 988036$

(d) $1005^2 = (1005 - 5)(1005 + 5) + 52 = 1010000 + 25 = 1010025$

Finding square using formula, for any two-digit numbers

$$(a+b)^2 = a^2 + 2ab + b^2$$



Example: 15

$$25^2$$

Simply, $a=2$ and $b=5$

a^2	$2ab$	b^2
4	20	25

 $4+2 \quad / \quad 2+20 \quad / \quad 5 = 625.$

Finding square using Reference method

Instructions:

- To find square for any number from 1^2 to 125^2 , we must know squares from 1^2 to 25^2 .
- If a square number falls between 26^2 to 75^2 , take 50 as a reference number.
- If a square number falls between 76^2 to 125^2 , take 100 as a reference number.

Taking 50 as reference,



Example: 16

49^2 , falls between 26^2 to 75^2 .

If then, 50 is the reference number.

Then do $50^2 = 25 \mid 00$

$50-1 = 49$. So, $-1 \mid (12)$

24 | 01

If the unit digit is a single digit number, then make zero before that number, (eg; 1 as 01)



Example: 17

39^2 , 50 as reference

$$50 - 11 = 39.$$

So, 25 | 00

$$-11 \mid (11)^2$$

$$14 \mid 121$$

Here, for 121 we have to take 21 only, remaining 1 will be carried for 14 to make as 15(1+14).

Hence, we have to concern only for last two places.

So, 1521.



Example: 18

59^2 ,

50 as reference

$$50 + 9 = 59$$

So, 25 | 00

$$+9 \mid 9^2$$

$$34 \mid 81$$

So, the answer is 3481.

Taking 100 as reference,



Example: 19

$$98^2 = 9604$$

Step 1: Find the difference between the given number and 100.

Step 2: Square the difference.

$$100 - 98 = 2.$$

So, $2^2 = 4$, but write it as 04 since it is a single digit number.

$$100 - 2 = 98, \text{ so subtract that 2 from 98.}$$

$$\text{Hence, } 98 - 2 = 96.$$

$$96 \mid 04 = 9604.$$



Example: 20

$$88^2 = 7744.$$

$$\text{Step 1: } 100 - 88 = 12$$

$$\text{Step 2: } 12^2 = 144.$$

So, take 44 and 1 as carry.

Then, the same 12 is to be subtracted from 88.

$$88-12= 76 +1(\text{carry})= 77.$$

$$77 \mid 44 = 7744.$$



Example: 21

$$105^2= 11025$$

$$100+5 =105$$

$$\text{So, } 5^2 = 25$$

Then the same 5 is to be added to 105 since it is in addition.

$$105+5 = 110$$

$$110 \mid 25 = 11025$$



Example: 22

$$112^2 =12544$$

$$100+12=112$$

$$\text{So, } 12^2 = 144.$$

Take 44 as last two digits and 1 as carry.

Then the same 12 is to be added to 112.

$$112+12+1(\text{Carry}) = 125$$

$$125 \mid 44 = 12544.$$

1.5 Common Factor

If a number a divides another number b exactly, we can say that a is a factor of b . In this case, b is called a multiple of a . A common factor of two or more numbers is a number which divides each of them exactly. For example, 4 is a common factor of 8 and 12.

1.6 Highest Common Factor

Highest common factor of two or more numbers is the greatest number that divides each one of them. Exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called Greatest Common Divisor or Greatest Common Measure. Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

1.7 Methods of Finding H.C.F.

I. Method of Prime Factors

Step1 Express each one of the given numbers as the product of prime factors. [A number is said to be a prime number if it is exactly divisible by 1 and itself, but not by any other number, e.g., 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose common factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.



Example 23: Find the H.C.F. of 70 and 90.

$$\text{Solution: } 70 = 2 \times 5 \times 7$$

$$90 = 2 \times 5 \times 9$$

Common factors are 2 and 5.

$$\therefore \text{H.C.F.} = 2 \times 5 = 10.$$



Example 24: Find the H.C.F. of 3332, 3724 and 4508.

$$\text{Solution: } 3332 = 2 \times 2 \times 7 \times 7 \times 17$$

$$3724 = 2 \times 2 \times 7 \times 7 \times 19$$

$$4508 = 2 \times 2 \times 7 \times 7 \times 23$$

$$\therefore \text{H.C.F.} = 2 \times 2 \times 7 \times 7 = 196.$$



Example 25: Find the H.C.F. of 360 and 132.

$$\text{Solution: } 360 = 2^3 \times 3^2 \times 5$$

$$132 = 2^2 \times 3^1 \times 11$$

$$\therefore \text{H.C.F.} = 2^2 \times 3^1 = 12.$$



Example 26: If $x = 2^3 \times 3^2 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y .

Solution: The factors common to both x and y are $2^3 \times 3^2 \times 5^9$

$$\therefore \text{H.C.F.} = 2^3 \times 3^2 \times 5^9$$

II. Method of Division

A. For two numbers:

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder. This is continued until no remainder is left.

H.C.F. is the divisor of last step.



Example 27: Find the H.C.F. of 3556 and 3444.

$$\begin{array}{r}
 3444 \overline{) 3556} \quad (1) \\
 \underline{-3444} \\
 112 \quad (30) \\
 \underline{-112} \\
 84 \quad (1) \\
 \underline{-84} \\
 28 \quad (3) \\
 \underline{-28} \\
 0 \quad \text{Reminder}
 \end{array}$$

HCF ← 28

A. For more than two numbers:

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Analytical Skills

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.



Example 28: Find the H.C.F. of 13915, 9499 and 2553 by division method.

Solution:

$$9499 \overline{)13915} (1$$

$$\underline{- 9499}$$

$$4416 \overline{)9499} (2$$

$$\underline{- 8832}$$

$$667 \overline{)4416} (6$$

$$\underline{- 4002}$$

$$414 \overline{)667} (1$$

$$\underline{- 414}$$

$$253 \overline{)414} (1$$

$$\underline{- 253}$$

$$161 \overline{)253} (1$$

$$\underline{- 161}$$

$$92 \overline{)161} (1$$

$$\underline{- 92}$$

$$69 \overline{)92} (1$$

$$\underline{- 69}$$

$$23 \overline{)69} (3$$

$$\underline{- 69}$$

$$0$$

Now, in the next step, we will find the H.C.F. of 23 and 2553.

$$23 \overline{)2553} (111$$

$$\underline{2553}$$

$$0$$

Thus, H.C.F. of 13915, 9499 and 2553 = 23.



Example 29: Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution: Required length

$$= (\text{H.C.F. of } 700, 385, 1295) \text{ cm} = 35 \text{ cm.}$$

Common multiple

A common multiple of two or more numbers is a number which is exactly divisible by each one of them. For example, 32 is a common multiple of 8 and 16.

$$8 \times 4 = 32$$

$$16 \times 2 = 32.$$

1.8 Least Common Multiple

The least common multiple of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 60, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

∴ Least common multiple, i.e., L.C.M. of 12 and 18 is 36.

Methods of Finding L.C.M.

Method of Prime Factors

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.



Example 30: Find the L.C.M. of 32, 48, 60 and 320.

Solution: $32 = 2^5 \times 1$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$320 = 2^6 \times 5$ ∴ L.C.M. = $2^6 \times 5 \times 3 = 960$.

II. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers, which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line, which is the required L.C.M.



Example 31: Find the L.C.M. of 12, 15, 20 and 54.

	12, 15, 20, 54
2	6, 15, 10, 27
2	3, 15, 5, 27
3	1, 5, 5, 9
5	1, 1, 1, 9

$$\text{L.C.M.} = 2 \times 2 \times 3 \times 5 \times 1 \times 1 \times 1 \times 9 = 540.$$

Notes

Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit

Short-cut methods

01 H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of Step 2) leaving as many digits on its right as there are in each of the numbers.



Example 32: Find the L.C.M. of 1.2, 0.24 and 6.

Solution: The given numbers can be written as 1.20, 0.24 and 6.00.

Now, ignoring the decimal we find the L.C.M. of 120, 24 and 600.

	120, 24, 600
2	60, 12, 300
2	30, 6, 150
2	15, 3, 75
3	5, 1, 25
5	1, 1, 5

$$\therefore \text{L.C.M.} = 2 \times 2 \times 2 \times 3 \times 5 \times 1 \times 5 = 600$$

Thus, the required L.C.M. = 6.00, i.e., 6.



Example 33: Find the H.C.F. of 6.16 and 13.

Solution: The given numbers can be written as 6.16 and 13.00.

Now, ignoring the decimals we find the H.C.F. of 616 and 1300.

616)1300(2

- 1232

68)616(9

- 612

4)68(17

-68

0

\therefore H.C.F. of 616 and 1300 is 4. Thus, the required H.C.F. = 0.04

2. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M.} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in the denominator}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in the denominator}}$$



Example 34: Find the L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$, and $\frac{6}{25}$

Solution: L.C.M. of numerators 2, 3 and 6 is 6.

H.C.F. of denominators 5, 10 and 25 is 5.

$$\therefore \text{Required L.C.M.} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in the denominator}} = \frac{6}{5}$$



Example 35: Find the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$, and $\frac{20}{63}$

Solution: H.C.F. of numerators 4, 10 and 20 is 2.

L.C.M. of denominators 9, 21 and 63 is 63.

$$\therefore \text{Required H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in the denominator}} = \frac{2}{63}$$

Notes

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.
2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

03 Product of two numbers = L.C.M. of the numbers \times H.C.F. of the numbers

Example 36: The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If

one of the two numbers is 315, find the other number.

Solution: The required number = $\frac{L.C.M \times H.C.F}{\text{First number}} = \frac{1260 \times 63}{315} = 252$

04 To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z.



Example 37: Find the greatest number that will exactly divide 200 and 320.

Solution: The required greatest number = H.C.F. of 200 and 320 = 40.

05 To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.



Example 38: Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution: The required greatest number = H.C.F. of $(148 - 4)$, $(246 - 6)$ and $(623 - 11)$,
i.e., H.C.F. of 144, 240 and 612 = 12.

06 To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z.



Example 39: What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution: The required smallest number = L.C.M. of 36, 45, 63 and 80 = 5040.

07 To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively.

It is always observed that $(x - a) = (y - b) = (z - c) = k$ (say)

\therefore Required number = (L.C.M. of x, y and z) - k.



Example 40: Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Solution: Since, $(36 - 25) = (48 - 37) = (64 - 53) = 11$, therefore, the required smallest number =
(L.C.M. of 36, 48 and 64) - 11 = 576 - 11 = 565.

08 To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) + r.



Example 41: Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution: The required smallest number = (L.C.M. of 12, 16 and 18) + 5 = 144 + 5 = 149.

09 To find the greatest number that will divide x, y and z leaving the same remainder in each case.

(a) When the value of remainder r is given: Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.

(b) When the value of remainder is not given: Required number = H.C.F. of $|x - y|$, $|y - z|$ and $|z - x|$



Example 42: Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution: The required greatest number = H.C.F. of $(772 - 5)$ and $(2778 - 5)$ = H.C.F. of 767 and 2773 = 59.



Example 43: Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

Solution: The required greatest number = H.C.F. of $|x - y|$, $|y - z|$ and $|z - x|$ =

H.C.F. of $|152 - 277|$, $|277 - 427|$ and $|427 - 152|$ = H.C.F. of 125, 150 and 275 = 25.

10 To find the n-digit greatest number which, when divided by x, y and z, leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and z = L

Step 2 $\frac{n \text{ digit greatest number}}{L}$, Remainder=R

Step 3 Required number = n-digit greatest number - R

(b) leaves remainder K in each case Required number = (n-digit greatest number - R) + K.



Example 44: Find the greatest number of 4-digit number which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252

252)9999(39

- 9828

171=Remainder (R)

∴ The required number = $(9999 - 171) + 3 = 9931$.



Example 45: Find the greatest number of four digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution: L.C.M. of 12, 15, 20 and 35 = 420.

420)9999(23

- 9660

339=Remainder (R)

∴ The required number = $9999 - 339 = 9663$.

11 To find the n-digit smallest number which when divided by x, y and z (a) leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and z = L

Step 2 $\frac{n \text{ digit smallest number}}{L}$, Remainder=R

Step 3 Required number = n-digit smallest number + (L - R).

(b) leaves remainder K in each case. Required number = n-digit smallest number + (L - R) + K.



Example 46: Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution: L.C.M. of 4, 6, 8 and 10 = 120.

120)1000(8

- 960

40=Remainder (R)

∴ The required number = $1000 + (120 - 40) = 1080$.



Example 47: Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252.

252)1000(3

- 756

244=Remainder (R)

∴ The required number = $1000 + (252 - 244) + 3 = 1011$.

Summary

The key concepts learned from this unit are: -

- We have learnt what are different types of numbers
- We have learnt Multiplication short cuts.
- We have learnt squaring short cuts.
- We have learnt Tests of Divisibility
- We have learnt Methods of Finding L.C.M.
- We have learnt Methods of Finding H.C.F.

Keywords

- Multiplication.
- Distributive Laws.
- Tests of Divisibility
- L.C.M.
- H.C.F.

Self Assessment

1. Which of the following statements is true?
 - A. 541326 is divisible by 3, 5967013 is not divisible by 3
 - B. 541326 is not divisible by 3, 5967013 is not divisible by 3
 - C. 541326 is divisible by 3, 5967013 is divisible by 3
 - D. 541326 is not divisible by 3, 5967013 is divisible by 3
2. Which of the following statements is true?
 - A. 67920594 is divisible by 4, 618703572 is not divisible by 4
 - B. 67920594 is not divisible by 4, 618703572 is not divisible by 4

- C. 67920594 is divisible by 4, 618703572 is divisible by 4
D. 67920594 is not divisible by 4, 618703572 is divisible by 4
3. 4832718 is divisible by?
A. 11
B. 5
C. 10
D. 25
4. Least number must be added to 3000 to obtain a number exactly divisible by 19 is?
A. 1
B. 2
C. 3
D. 4
5. The H.C.F. of 108, 288 and 360 is?
A. 24
B. 28
C. 36
D. 48
6. The H.C.F. of 513, 1134 and 1215 is?
A. 13
B. 27
C. 81
D. 124
7. The L.C.M. of 72, 108 and 2100 is?
A. 38700
B. 32500
C. 37800
D. 32
8. The L.C.M. of 16, 24, 36 and 54 is?
A. 432
B. 411
C. 32
D. 24
9. The H.C.F. and L.C.M. of 0.63, 1.05 and 2.1 is?
A. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.33, 1.05 and 2.1 is 5
B. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.93, 1.05 and 2.1 is 6.30
C. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.63, 1.05 and 2.1 is 6.30
D. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.63, 1.05 and 2.1 is 9.30
10. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, then the numbers are?
A. 195 and 200.
B. 200 and 143.

- C. 195 and 145.
D. 195 and 143.
11. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, then the other is?
A. 95
B. 99
C. 120
D. 200
12. A number when successively divide by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.
A. 5, 4, 2
B. 6, 4, 2
C. 1, 1, 3
D. None of these
13. Find the remainder when 25^{102} is divided by 17.
A. 5
B. 6
C. 4
D. None of these
14. What is the largest number which divides 97 and 65 leaving remainders of 7 and 5 respectively?
A. 60
B. 30
C. 15
D. 10
15. What is the largest number that divides 23, 37 and 79 leaving the same remainder in each case?
A. 2
B. 7
C. 12
D. 14

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. A | 2. D | 3. A | 4. B | 5. C |
| 6. B | 7. C | 8. A | 9. D | 10. D |
| 11. B | 12. D | 13. C | 14. B | 15. D |

Review Questions

1. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.
2. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.
3. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case
4. Find the least number exactly divisible by 12,15,20,27.
5. Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case.
6. The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec., respectively. If they all change simultaneously at 8:20:00hours, then at what time they again change simultaneously.
7. A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.
8. Find the least number of 5 digits which is exactly divisible by 632.
9. Find the greatest number of 5 digits which is exactly divisible by 463.
10. Find the number nearest to 13700 which is exactly divisible by 235.

**Further Readings**

- Quantitative Aptitude for Competitive Examinations, By Dr. R S Aggarwal, S Chand Publishing
- A Modern Approach, To Verbal & Non-Verbal Reasoning, By Dr. R S Aggarwal, S Chand Publishing
- Magical Book on Quicker Maths, By M Tyra, Banking Service Chronicle
- 4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Unit 02: Average

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Objectives

Introduction

2.1 Average

2.2 Weighted Average

2.3 Short Cut Methods

Summary

Keywords

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- The average of all numbers and Arithmetic mean.
- Weighted Average.
- Application based questions of Average.

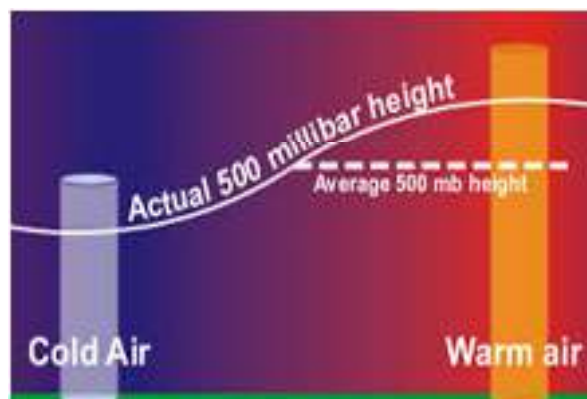
Introduction

What is an average??

Average of n values is equal to the sum of n values divided by the total divide by number of values (n)

Averages = Sum of Observations / Number of Observation

$$\text{Avg} = \text{Sum}/n$$



Numbers are an integral part of our everyday lives, right from the number of hours we sleep at night

Some Properties of Average:

Analytical Skills

- If we have N number of observation as $a_1, a_2, a_3, a_4, \dots, a_N$ and their average is A, then
- If we add or subtract the same value (suppose x) from all the observations then average will be directly added or subtracted by x.
- If we multiply or divide each observation by a same value (Suppose x), then average also multiplied or divided by the same value x.

If x is the average of x_1, x_2, \dots, x_n , then

- (a) The averages of $x_1 + a, x_2 + a, \dots, x_n + a$ is $x + a$.
- (b) The averages of $x_1 - a, x_2 - a, \dots, x_n - a$ is $x - a$.
- (c) The average of ax_1, ax_2, \dots, ax_n is ax .
- (d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \dots, \frac{x_n}{a}$ is $\frac{x}{a}$, provided $a \neq 0$.

If terms are in A.P or Consecutives:

- If the terms are consecutives or are in A.P then average is always the middle term of the sequence if the number of observations is odd.
- If number of observations are even then average is the average of two middle numbers.

Note:

Average of consecutive numbers can also be written as

(1st term + last term)/2 or (2nd term + 2nd last term)/2 or (3rd term + 3rd last term)/2 and so on.

Deviation Method:



Example: Find the average of 20, 23, 28, 24, 25

Assume any number as the average, say 25.

Find the deviation for all the given values

Numbers : 20 23 28 24 25

Deviation : -5 -2 +3 -1 0

Total deviation : -5

Average deviation = $-5/5$

(total/n) = -1

Final average = assumed average + average deviation

= $25 + (-1) = 24$



If we have to ask to find the average of all prime numbers between 30 and 50?

Sol: There are five prime numbers between 30 and 50.

They are 31, 37, 41, 43 and 47.

Now we can find the average by any of the method either by deviation method or conventional method of finding average.

The required average = $(31+37+41+43+47)/5 = 199/5 = 39.8$.

Whenever we are asked the marks we scored in any examination, we usually tell the marks in terms of percentage that is, taking the percentage of total marks of all subjects. This percentage is called average percentage. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about their average age. Introduction Whenever we are asked

the marks we scored in any examination, we usually tell the marks in terms of percentage, that is, taking the percentage of total marks of all subjects. This percentage is called average percentage. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about their average age.

$$\frac{3 + 9 + 11 + 15 + 18 + 19 + 23}{7} = \frac{98}{7} = 14$$

2.1 Average

$$01 \text{ Average} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$$

$$02 \text{ Sum of quantities} = \text{Average} \times \text{Number of quantities}$$

$$03 \text{ Number of quantities} = \frac{\text{sum of quantities}}{\text{Average}}$$



Find the average of first 40 natural numbers?

Sol: sum of first n natural numbers = $n(n+1)/2$; So, sum of 40 natural numbers = $(40 \times 41)/2 = 820$.

Therefore, the required average = $(820/40) = 20.5$.

Alternatively: Since Natural numbers are in consecutive term or in A.P.

Therefore, Average can also be obtained as

$$\text{Average} = (\text{1st term} + \text{last term})/2 = (1+40)/2 = 41/2 = 20.5.$$



Example 1: A man purchased 5 toys at Rs.200 each, 6 toys at Rs. 250 each and 9 toys at Rs. 300 each.

Calculate the average cost of 1 toy.

Solution: Price of 5 toys = $200 \times 5 = \text{Rs.}1000$

Price of 6 toys = $250 \times 6 = \text{Rs.}1500$

Price of 9 toys = $300 \times 9 = \text{Rs.}2700$

Total number of toys = $5 + 6 + 9 = 20$

$$\text{Average price of 1 toy} = \frac{1000+1500+2700}{20} = \frac{5200}{20} = \text{Rs.}260.$$



Example 2: Find the average of first 20 multiples of 7?

Solution: Required average = $7(1+2+3+\dots+20)/20 = (7 \times 20 \times 21)/(20 \times 2) = (147/2) = 73.5$.

Alternatively: Since Multiples of 7 are in A.P.

Therefore, Average can also be obtained as

$$\text{Average} = (\text{1st term} + \text{last term})/2 = (7 + 140)/2 = 147/2 = 73.5.$$



Example 3: The average marks obtained by 200 students in a certain examination is 45. Find the total marks.

Solution: Total marks

Analytical Skills

= Average marks \times Number of students

$$= 200 \times 45 = 900.$$



Example 4: The average of four consecutive even numbers is 27. find the largest of these numbers?

Solution: let the numbers be x , $x+2$, $x+4$ and $x+6$. then,

$$(x + (x+2) + (x+4) + (x+6))/4 = 27$$

$$\text{Or } (4x+12)/4 = 27$$

$$\text{Or } x+3 = 27$$

$$\text{Or } x = 24.$$

Therefore, the largest number $= (x+6) = 24+6 = 30$.

Alternatively:

$$\text{Average of consecutive terms} = (\text{1st term} + \text{last term})/2 = (x + (x+6))/2 = 27$$

$$\text{Or } x+3 = 27$$

$$\text{Or } x = 24.$$

Therefore, the largest number $= (x+6) = 24+6 = 30$.



Example 5: Total temperature for the month of September is 840°C . If the average temperature of that month is 28°C , find out the number of days in the month of September.

Solution: Number of days in the month of September

$$= \frac{\text{Total temperature}}{\text{Average temperature}} = \frac{840}{28} = 30 \text{ days}$$

2.2 Weighted Average

To find the weighted average (or weighted mean) of a data set, multiply each value by its weight and add the products together, then divide by the sum of the weights.

For a set of data x_1, x_2, \dots, x_n , with nonnegative weights w_1, w_2, \dots, w_n ,

The weighted average is

$$\frac{x_1 w_1 + x_2 w_2 + \dots + x_n w_n}{w_1 + w_2 + \dots + w_n}$$

Finding a Weighted Average



Example 6: A grade point average is a weighted average that gives greater weight to courses that earn more credits. Hailey's grade points are 4.0 in Chemistry, which is worth 4 credits, 3.5 in

English, which is worth 3 credits, and 3.7 in Physics, which is worth 2 credits. What is Hailey's grade point average?

Solution: Multiply each grade by its weight. Add the products and then divide by the sum of the weights.

$$\frac{4.0 \times 4 + 3.5 \times 3 + 3.7 \times 2}{4 + 3 + 2} = \frac{33.9}{9} = 3.7666 \approx 3.77$$

Hailey's grade point average is approximately 3.77.

2.3 Short Cut Methods

- Average of two or more groups taken together. (a) If the number of quantities in two groups are n_1 and n_2 and their average is x and y , respectively, the combined average (average of all of them put together) is

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

Explanation

Number of quantities in the first group = n_1 , and Their average = x

$$\therefore \text{Sum} = n_1 \times x$$

Number of quantities in the second group = n_2 , and Their average = y

$$\therefore \text{Sum} = n_2 \times y$$

Number of quantities in the combined group = $n_1 + n_2$.

Total sum (sum of quantities of the first group and the second group) = $n_1x + n_2y$.

\therefore Average of the two groups is

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

(b) If the average of n_1 quantities is x , and the average of n_2 quantities out of them is y , the average of the remaining group (rest of the quantities) is

$$\frac{n_1x - n_2y}{n_1 - n_2}$$

Explanation

Number of quantities = n_1 and Their average = x

$$\therefore \text{Sum} = n_1 \times x$$

Number of quantities taken out = n_2 and Their average = y

$$\therefore \text{Sum} = n_2y$$

Sum of remaining quantities = $n_1x - n_2y$, and Number of remaining quantities = $n_1 - n_2$

\therefore Average of the remaining group

$$= \frac{n_1x - n_2y}{n_1 - n_2}$$



Example 7: There are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40kg and that of section B is 35kg, find the average weight of the whole class?

Solution: Total weight of (36+44) students = $(36 \times 40 + 44 \times 35)$ kg = 2980kg.

Therefore, weight of the total class = $(2980/80)$ kg = 37.25kg.



Example 8: The average weight of 24 students of section A of a class is 58 Kg, whereas the average weight of 26 students of section B of the same class is 60.5 Kg. Find out average weight of all the 50 students of the class.

Solution: Here, $n_1 = 24$, $n_2 = 26$, $x = 58$, and $y = 60.5$.

\therefore Average weight of all the 50 students

$$\frac{n_1x + n_2y}{n_1 + n_2} = \frac{24 \times 58 + 26 \times 60.5}{24 + 26} = 59.3 \text{ kg}$$



Example 9: Average salary of all the 50 employees including 5 officers of a company is rs.850. If the average salary of the officers is rs.2500, find the average salary of the remaining staff of the company.

Solution: Here, $n_1 = 50$, $n_2 = 5$, $x = 850$ and $y = 2500$.

\therefore Average salary of the remaining staff

$$= \frac{n_1x - n_2y}{n_1 - n_2} = \frac{50 \times 850 - 5 \times 2500}{50 - 5} = 666.6666 \approx \text{Rs. } 667 \text{ (approx.)}$$



Example 10: The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Solution: The new average = $\bar{x} + a = 19 + 8 = 27$



Example 11: The average of some numbers is $5x$. If $x - 2$ is subtracted from each given number, what will be the new average?

Solution: The new average = $\bar{x} - a = 5x - (x - 2) = 4x + 2$.



Example 12: The average of 8 numbers is 21. If each of the numbers is multiplied by 8, find the average of a new set of numbers.

Solution: The average of a new set of numbers = $a\bar{x} = 8 \times 21 = 168$

- The average of n quantities is equal to x . If one of the given quantities whose value is p , is replaced by a new quantity having value q , the average becomes y ,
- then $q = p + n(y - x)$.



Example 13: The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.

Solution: Total weight increased = (1.8×10) kg = 18 kg.

\therefore Weight of the new man = $(53 + 18)$ kg = 71 kg



Example 14: The average of 25 result is 18. The average of 1st 12 of them is 14 & that of last 12 is 17. Find the 13th result.

Solution: Clearly 13th result = (sum of 25 results) - (sum of 24 results) = $(18 \times 25) - (14 \times 12) + (17 \times 12)$
 $= 450 - (168 + 204) = 450 - 372 = 78$



Example 15: The average weight of 25 persons is increased by 2 Kg when one of them whose weight is 60 Kg, is replaced by a new person. What is the weight of the new person?

Solution: The weight of the new person

$$= p + n(y - x)$$

$$= 60 + 25(2) = 110 \text{ Kg.}$$

- The average of n quantities is equal to x . When a quantity is removed, the average becomes y . The value of the removed quantity is $n(x - y) + y$.
- The average of n quantities is equal to y . When a quantity is added, the average becomes x . The value of the new quantity is $n(y - x) + y$.



Example 16: The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year. What is the age of the class teacher?

Solution: The age of class teacher = $n(x - y) + y$

$$= 25(16 - 15) + 15 = 40 \text{ years.}$$



Example 17: The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacher's age

Solution: The teacher's age = $n(y - x) + y$

$$= 30(10 - 9) + 10 = 40 \text{ years.}$$



Example 18: A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by 3. Find his average after 17th inning.

Solution: Let the average after 17th inning = x . Then, average after 16th inning = $(x - 3)$.

$$\therefore 16(x - 3) + 87 = 17x \text{ or } x = (87 - 48) = 39$$



Example 19: Nine persons went to a hotel for taking their meals 8 of them spent Rs.12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine. What was the total money spent by them?

Solution: Let the average expenditure of all nine be Rs. x

Then $12 \times 8 + (x+8) = 9x$ or $8x = 104$ or $x = 13$.

Total money spent = $9x = \text{Rs. } (9 \times 13) = \text{Rs. } 117$

- The average of first n natural numbers is $\frac{n+1}{2}$ or Average = $(1^{\text{st}} \text{ term} + \text{last term})/2$
- The average of square of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$
- The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$
- The average of odd numbers from 1 to n is $\frac{\text{last odd number} + 1}{2}$
- The average of even numbers from 1 to n is $\frac{\text{last even number} + 2}{2}$



Example 20: Find the average of first 81 natural numbers.

Solution: The average of first n natural numbers is $\frac{n+1}{2}$ or Average = $(1^{\text{st}} \text{ term} + \text{last term})/2$

$$= \frac{81 + 1}{2} = 41$$



Example 21: What is the average of squares of the natural numbers from 1 to 41?

Solution: The average of square of natural numbers till n is

$$\frac{(n+1)(2n+1)}{6} = \frac{(41+1)(2 \times 41 + 1)}{6} = 581$$



Example 22: Find the average of cubes of natural numbers from 1 to 27.

Solution: The average of cubes of natural numbers till n is

$$\frac{n(n+1)^2}{4} = \frac{27(27+1)^2}{4} = 5292$$



Example 23: What is the average of odd numbers from 1 to 40?

Solution: The required average = $\frac{\text{last odd number} + 1}{2} = \frac{39 + 1}{2} = 20$



Example 24: What is the average of even numbers from 1 to 81?

Solution: The required average = $\frac{\text{last even number} + 2}{2} = \frac{80 + 2}{2} = 41$

- If a person covers certain distance with speed x km/h and return back with speed y km/h then his average speed throughout the journey is

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{D+D}{\frac{D}{x} + \frac{D}{y}} = \frac{2xy}{x+y} \text{ km/h}$$



Example 25: Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km per hour. Find the average speed of the train during the whole journey.

Solution: The required average speed = $\frac{(2xy)/(x+y))\text{km/hr}}{(2 \times 84 \times 56)/(84+56)\text{km/hr}} = \frac{(2 \times 84 \times 56)}{140} \text{ km/hr} = 67.2 \text{ km/hr}$.

- If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.
- If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.
- The averages of first n consecutive even numbers is $(n + 1)$.
- The average of first n consecutive odd numbers is n .
- The averages of squares of first n consecutive even numbers is $\frac{2(n+1)(2n+1)}{3}$
- The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$
- The average of squares of consecutive odd numbers till n is $\frac{n(n+2)}{3}$
- If the average of n consecutive numbers is m , then the difference between the smallest and the largest number is $2(m - 1)$.



Example 26: Find the average of 7 consecutive numbers 3, 4, 5, 6, 7, 8, 9.

Solution: The required average = middle number = 6.



Example 27: Find the average of consecutive odd numbers 21, 23, 25, 27, 29, 31, 33, 35.

Solution: The required average = average of middle two numbers

$$= \text{average of 27 and 29} = \frac{27 + 29}{2} = 28$$



Example 28: Find the average of first 31 consecutive even numbers.

Solution: The required average = $(n + 1) = 31 + 1 = 32$.



Example 29: Find the average of first 50 consecutive odd numbers.

Solution: The required average = $n = 50$.



Example 30: Find the average of squares of first 19 consecutive even numbers.

$$\text{Solution: The required average} = \frac{2(n+1)(2n+1)}{3} = \frac{2(19+1)(2 \times 19+1)}{3} = \frac{2 \times 20 \times 39}{3} = 520$$



Example 31: Find the average of squares of consecutive even numbers from 1 to 25.

Solution: The required average

Analytical Skills

$$= \frac{(n+1)(n+2)}{3} = \frac{(25+1)(25+2)}{3} = 234$$



Example 32: Find the average of squares of consecutive odd numbers from 1 to 31.

Solution: The required average

$$= \frac{n(n+2)}{3} = \frac{31(31+2)}{3} = 341$$



Example 33: If the average of 6 consecutive numbers is 48, what is the difference between the smallest and the largest number?

Solution: The required difference

$$= 2(n-1) = 2(6-1) = 10.$$

Geometric Mean or Geometric Average

Geometric mean of x_1, x_2, \dots, x_n is denoted by

$$G.M = \sqrt[n]{x_1 \times x_2 \times \dots \times x_n}$$

- Geometric mean is useful in calculating averages of ratios such as average population growth rate, average percentage increase and, so on.



Example 34: The production of a company for three successive years has increased by 10%, 20% and 40%, respectively. What is the average annual increase of production?

Solution: Geometric mean of x, y and $z = \sqrt[3]{(x \times y \times z)}$

$$\therefore \text{Average increase} = (10 \times 20 \times 40)^{1/3} \% = 20\%$$



Example 35: The population of a city in two successive years increases at the rates of 16% and 4%, respectively. Find out the average increase in two years.

Solution: In case of population increase, the geometric mean is required.

\therefore Geometric mean of 16% and 4% is

$$= (16 \times 4)^{1/2} \%, \text{ i.e., } 8\%$$

Harmonic Mean or Harmonic Average.

Harmonic mean of x_1, x_2, \dots, x_n is denoted by

$$H.M = \frac{1}{\frac{1}{n} \left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

Harmonic mean is useful in finding out average speed of a vehicle, average production per day and, so on.



Example 36: A man runs 1 Km at a speed of 15 Km/h and another 1 Km he walks at a speed of

5 Km/h. Find out his average speed in covering 2 Km.

Solution: Harmonic mean is used when distance remains constant and speed varies. Harmonic

mean of x and y is $\frac{2}{\frac{1}{x} + \frac{1}{y}} = \frac{2xy}{x+y}$

\therefore Average speed for the whole distance

$$= \frac{2 \times 15 \times 5}{15+5} = 7.5 \text{ km/h}$$



Example 37: If half of the journey is travelled at a speed of 15 Km/h and the remaining half at a speed of 12 Km/h, find out average speed during the entire journey

Solution: The average speed

$$= \frac{2xy}{x+y} = \frac{2 \times 15 \times 12}{15+12} = \frac{40}{3} = 13\frac{1}{3} \text{ km/h}$$



Example 38: A man goes to a certain place at a speed of 30 Km/h and returns to the original place at a speed of 20 Km/h, find out his average speed during this up and down journey.

Solution: The average speed

$$= \frac{2xy}{x+y} = \frac{2 \times 30 \times 20}{30+20} = 24 \text{ km/h}$$

- If a person or a motor car covers three equal distances at the speed of x Km/h, y Km/h and z Km/h, respectively, then for the entire journey average speed of the person or motor car is $= \frac{3xyz}{xy+yz+zx}$



Example 39: A train covers the first 160 Km at a speed of 120 Km/h, another 160 Km at 140

Km/h and the last 160 Kms at 80 Km/h. Find out average speed of the train for the entire journey.

$$\text{Solution: Average speed} = \frac{3xyz}{xy+yz+zx} = \frac{3 \times 120 \times 140 \times 80}{120 \times 140 + 140 \times 80 + 80 \times 120} = \frac{4032000}{37600} = 107\frac{11}{47} \text{ km/h}$$

- If a person covers A Km at a speed of X Km/h, B Km at a speed of Y Km/h and C Km at a speed of Z Km/h, the average speed during the entire journey is

$$= \left(\frac{A+B+C}{\frac{A}{X} + \frac{B}{Y} + \frac{C}{Z}} \right) \text{ km/h}$$



Example 40: A person covers 9 Km at a speed of 3 Km/h, 25 Km at a speed of 5 Km/h and 30

Km at a speed of 10 Km/h. Find out average speed for the entire journey.

$$\text{Solution: The average speed} = \left(\frac{A+B+C}{\frac{A}{X} + \frac{B}{Y} + \frac{C}{Z}} \right) \text{ km/h} = \left(\frac{9+25+30}{\frac{9}{3} + \frac{25}{5} + \frac{30}{10}} \right) = \frac{64}{11} = 5\frac{9}{11} \text{ km/h}$$

- If a person covers A th part of the distance at X Km/h, Bth part of the distance at Y Km/h and the remaining C th part at Z Km/h, then the average speed during the entire journey is

$$= \left(\frac{1}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}} \right) \text{ km/h}$$



Example 41: A person covers the first $\frac{1}{4}$ of the distance at 8 Km/h, the next $\frac{3}{5}$ at 6 Km/h and the

remaining distance at 15 Km/h. Find the average speed during the entire journey.

$$\text{Solution: The average speed} = \left(\frac{1}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}} \right) = \left(\frac{1}{\frac{1/4}{8} + \frac{3/5}{6} + \frac{3/20}{15}} \right) = \left(\frac{1}{\frac{1}{32} + \frac{1}{10} + \frac{1}{100}} \right) = \frac{3200}{452} = \frac{800}{113} = 113 \frac{9}{113} \text{ km/h}$$

[Here, $A = 1/4$, $B = 3/5$, and $C = 1 - 1/4 - 3/5 = 3/20$]



Example 42: A train covers 50% of the journey at 30 Km/h, 25% of the journey at 25 Km/h and the remaining at 20 Km/h. Find the average speed of the train during the entire journey.

$$\text{Solution: The average speed} = \left(\frac{1}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}} \right) = \left(\frac{1}{\frac{1/2}{30} + \frac{1/4}{25} + \frac{1/4}{20}} \right) = \left(\frac{1}{\frac{1}{60} + \frac{1}{100} + \frac{1}{80}} \right) = \frac{2400}{94} = \frac{1200}{47} = 25 \frac{25}{47} \text{ km/h}$$

[Here, $A = 50\% = 1/2$, $B = 25\% = 1/4$, and $C = 1 - 1/2 - 1/4 = 1/4$]

Summary

The key concepts learned from this unit are: -

- We have learnt the average of all numbers.
- We have learnt about Weighted Average.

Keywords

- average
- Weighted Average.

Self Assessment

1. The average of 10 numbers is 7. What will be the new average if each of the numbers is multiplied by 8?

- A. 45
- B. 52
- C. 56
- D. 55

2. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by Rs42 per day while the average expenditure per head decreased by Rs 1. Find out the actual expenditure of the mess.

- A. Rs 480
- B. Rs 440
- C. Rs 520
- D. Rs 420

3. An aero plane travels 2500 Km, 1200 Km and 500 Km at 500 Km/h, 400 Km, and 250 Km/h, respectively. The average speed is:

- A. 420 Km/h
- B. 410 Km/h
- C. 405 Km/h
- D. 575 Km/h

4. The average age of 5 members is 21 years. If the age of the youngest member be 5 years, find out the average age of the family at the birth of the youngest member.

- A. 24 years
- B. 25 years
- C. 20 years
- D. 28 years

5. The average weight of 10 students is increased by half a Kg when one of the students weighing 50 Kg is replaced by a new student. Find out the weight of the new student.

- A. 55 Kg
- B. 60 Kg
- C. 45 Kg
- D. 40 Kg

6. The mean marks of 10 boys in a class is 70%, whereas the mean marks of 15 girls is 60%. The mean marks of all the 25 students is:

- A. 64%
- B. 60%
- C. 55%
- D. 52%

7. The average of five consecutive even numbers starting with 4, is:

- A. 6
- B. 7
- C. 8
- D. 7.5

Analytical Skills

8. The average of 17 numbers is 10.9. If the average of first nine numbers is 10.5 and that of the last 9 numbers is 11.4, the middle number is:

- A. 11.8
- B. 11.4
- C. 10.9
- D. 11.7

9. The sum of three numbers is 98. If the ratio between first and second be 2:3 and between second and third be 5:8, then the second number is:

- A. 30
- B. 20
- C. 58
- D. 48

10. The average age of A, B, C, D five years ago was 45 years. By including x, the present average age of all the five is 49 years. The present age of x is:

- A. 64 years
- B. 48 years
- C. 45 years
- D. 40 years

11. The average age of 8 men is increased by 2 years. When 2 of them, whose ages are 20 years and 24 years respectively, are replaced by 2 women. What is the average age of these two women?

- A. 36 years
- B. 30 years
- C. 40 years
- D. 42 years

12. At a language school, each student is given a score that measures his or her fluency. The fluency score is a weighted average that is determined by rating the student on a scale of 0 to 10 in three categories: grammar, vocabulary, and pronunciation. Grammar counts for 40% of the score, vocabulary counts for 25%, and pronunciation counts for 35%. Thomas gets ratings of 8 for grammar, 6 for vocabulary, and 5 for pronunciation. What is his fluency score?

- A. 7.45
- B. 6.45
- C. 7.50
- D. 7.70

13. A grade point average is a weighted average that gives greater weight to courses that earn more credits. Hailey's grade points are 4.0 in Chemistry, which is worth 4 credits, 3.5 in English, which is worth 3 credits, and 3.7 in Physics, which is worth 2 credits. What is Hailey's grade point average?

- A. 8.65
- B. 3.76
- C. 9.88

D. 7.70

14. The 30 students in Ms. Chen's class had an average grade of 85 on a standardized test. The 20 students in Mr. Jackson's class had an average grade of 90 on the same test. What is the average test grade of all 50 students?

A. 80.0

B. 60

C. 19.5

D. 67.20

15. A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by. Find his average after 17th inning.

A. 39

B. 40

C. 42

D. 69

Answers for Self Assessment

1. C 2. D 3. A 4. C 5. A

6. A 7. C 8. B 9. A 10. C

11. B 12. B 13. B 14. D 15. A

Review Questions

1. There were 35 students in a hostel. Due to the admission of 7 new students, he expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Rs 1. What was the original expenditure of the mess?

2. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the Newman.

3. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

4. The average weight of A, B, C is 45 Kg. The avg wt. of A & B be 40Kg & that of B & C be 43Kg. Find the wt. of B.

5. Find the average of first 40 natural numbers?

6. A tourist covers half of this journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. What is the average speed of the tourist in km/h during his entire journey.

7. What is the average of all multiples of 10 from 2 to 198?

8. The average weight of 16 boys in a class is 50.25 kg and that of the remaining 8 boys is 45.15 kg. Find the average weights of all the boys in the class.

9. In what ratio must water be mixed with milk to gain 16 2/3% on selling the mixture at cost price?

10. In what ratio must a grocer mix two varieties of pulses costing H15 and H20 per kg respectively so as to get a mixture worth H16.50 kg?



Further Readings

1. Quantitative Aptitude for Competitive Examinations, By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach, To Verbal & Non-Verbal Reasoning, By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book on Quicker Maths, By M Tyra, Banking Service Chronicle
4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Unit 03: Number Series

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Summary

Keywords

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Answers for Self Assessment

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Further Readings

Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- What is sequence and series?
- What are the different types of series?
- Identify the pattern in the given series.
- Solve different problems based on series completion.
- What is analogy?
- Solve problems based on Analogy and Classification of Number series.

Introduction

In almost all competitive exams, problems related to sequence and series are asked. For example, A numerical series is given in which a number is wrongly placed and asked to identify that particular wrong number. A numerical series is given in which a specific number is missing and asked to find out that missing number. A complete numerical series is followed by an incomplete numerical series, and we need to solve that incomplete numerical series in the same pattern in which the complete numerical series is given.

3.1 Sequence

It is a set of numbers, follows a definite order/rule. Let us consider a sequence $a_1, a_2, a_3, a_4, \dots, a_n$; where n is positive integer. For example, 1, 3, 5, 7,, 21 or 2, 4, 6, 8, 10,, etc. A sequence may be finite or infinite. It is represented by $\{a_n\}$

3.2 Series

A series is a sequence of numbers/letters. These numbers/letters are called terms of the sequence. All the terms of the sequence are arranged according to a certain predefined rule. After carefully studying the given series and finding the specific pattern in which the terms are changing, it is possible to find out the next term of the series.

Series may be classified as –

- Number Series
- Alphabetical/Letter Series
- Alpha-Numeric Series
- Special Series
- Pattern Series

3.3 Number Series

- A number series is given in which a number is wrongly placed. You are asked to identify that particular wrong number.
- A numerical series is given in which a specific number is missing. You are required to find out that missing number.
- A complete numerical series is followed by an incomplete numerical series. You need to solve that incomplete numerical series in the same pattern in which the complete numerical series is given.

Types of Number Series

Number series may be classified as –

1. Arithmetic Series
2. Arithmetic Series – Second Order
3. Arithmetic Series – Third Order
4. Geometric Series
5. Arithmetic-Geometric Series
6. Geometric-Arithmetic Series
7. Series of Squares, Cubes,
8. Double Series
9. Special Series

1. Arithmetic Series

An arithmetic series is one in which the difference between any two consecutive terms is always the same and is called the common difference. The successive number is obtained by using the formula - Successive Number = Previous Number (+) or (-) Fixed Number.



Example: 1, 4, 7, 10, 13,

- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \dots = 3$ (Common Difference)



Example: 13, 10, 7, 4, 1,

- $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \dots = -3$ (Common Difference)

2. Arithmetic Series - Second Order

A series in which the difference between successive terms themselves form an arithmetic series is called an arithmetic series of second order.



Example: 1, 3, 7, 13,

- $T_2 - T_1 = 3 - 1 = 2$
- $T_3 - T_2 = 7 - 3 = 4$
- $T_4 - T_3 = 13 - 7 = 6$

Difference in series: 2, 4, 6, ...

- $4 - 2 = 6 - 4 = \dots = 2$ (Common Difference)

Therefore, $T_5 = 13 + 8 = 21$

3. Arithmetic Series - Third Order

A series in which the difference between successive terms themselves form an arithmetic series of second order, is called an arithmetic series of third order.



Example: 2, 9, 17, 28, 44,

- $T_2 - T_1 = 9 - 2 = 7; T_3 - T_2 = 17 - 9 = 8; T_4 - T_3 = 28 - 17 = 11; T_5 - T_4 = 44 - 28 = 16$

Difference in series: 7, 8, 11, 16, ...

- $8 - 7 = 1; 11 - 8 = 3; 16 - 11 = 5$
- 1, 3, 5,
- $3 - 1 = 5 - 3 = \dots = 2$ (Common Difference)
- $T_6 = (44 + 16 + 7) = 67$

4. Geometric Series

A geometric series is one in which the ratio of any two consecutive terms is always the same and is called the common ratio. Each successive number is by using the formula - Successive Number = Previous Number (x) or (÷) Fixed Number.



Example: 4, 8, 16, 32,

- $\frac{8}{4} = \frac{16}{8} = \frac{32}{16} = \dots = 2$ (Common Ratio)



Example: 64, 32, 16, 8,

- $\frac{32}{64} = \frac{16}{32} = \frac{8}{16} = \dots = \frac{1}{2}$ (Common Ratio)

5. Arithmetic-Geometric Series

The given series is arithmetic-geometric series if each successive term is obtained by using the formula rule - Successive Term = (Previous term + Fixed Number) × (Another Fixed Number).



Example: 1, 9, 33, 105, ...

- $T_2 - T_1 = 9 - 1 = 8; T_3 - T_2 = 33 - 9 = 24; T_4 - T_3 = 105 - 33 = 72$
- 8, 24, 72, ... (Geometric Series)
- $T_2 = 9 = (1 + 2) \times 3; T_3 = 33 = (9 + 2) \times 3; T_4 = 105 = (33 + 2) \times 3$

- $T_5 = (105 + 2) \times 3 = 107 \times 3 = 321$

6. Geometric-Arithmetic Series

The given series is geometric-arithmetic series if each successive term is obtained by using the formula rule - Successive Term = (Previous term (x)/(÷) Fixed Number) (+)/(-) (Another Fixed Number).



Example: 2, 5, 17, 65, ...

- $T_2 - T_1 = 5 - 2 = 3; T_3 - T_2 = 17 - 5 = 12; T_4 - T_3 = 65 - 17 = 48$
- 3, 12, 48, ... (Geometric Series, Common Ratio = 4)
- $T_2 = 5 = (2 \times 4) - 3; T_3 = 17 = (5 \times 4) - 3; T_4 = 65 = (17 \times 4) - 3$
- $T_5 = (65 \times 4) - 3 = 260 - 3 = 257$

7. Series of Squares, Cubes

Series of squares, cubes, etc. are simple powers of natural numbers i.e., squares, cubes, or their combinations.



Example: 4, 9, 16, 25, 36, 49, 64,

- $2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, \dots$
- 2, 3, 4, 5, 6, 7, 8,
- Common Difference = 1
- $T_8 = (8 + 1)2 = (9)2 = 81$



Example: $\frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \frac{25}{216}, \dots$

- $\frac{1^2}{2^3}, \frac{2^2}{3^3}, \frac{3^2}{4^3}, \frac{4^2}{5^3}, \frac{5^2}{6^3}, \dots$
- $\frac{n^2}{(n+1)^3}$
- $T_6 = \frac{6^2}{7^3} = \frac{216}{343}$

8. Double Series

It consists of two series combined into a single series. The alternating terms of this series form an independent series



Example: 1, 2, 4, 6, 7, 18, 10, 54,

- 1, 4, 7, 10, - Arithmetic Series
- Common Difference = $4 - 1 = 7 - 4 = 10 - 7 = 3$
- 2, 6, 18, 54, - Geometric Series
- Common Ratio = $\frac{6}{2}, \frac{18}{6}, \frac{54}{18}, \dots = 3$
- Next Term = 13 (i.e., $10 + 3$)

9. Special Series

The special series may be classified as -

- Series of Date/Time
- Numbers followed by their L.C.M. or H.C.F
- Numbers Followed by their Product
- By Use of Digit Sum

Series of Date/Time



Example:

- 3 - 9 - 2022, 13 - 9 - 2022, 23 - 9 - 2022, & 2 - 10 - 2022.
- Differs by 10 days.
- September, 2022 - 30 days
- 2 - 10 - 2022 replaced by 3 - 10 - 2022



Example:

- 1:00, 2:25, 3:40, 5:15, 6:40
- Difference - 1 hour 25 min. • 3:40 should be replaced by 3:50.

Numbers Followed by Their L.C.M. or H.C.F



Example:

1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ?

- L.C.M. of 1, 2 & 3 = 6
- L.C.M. of 4, 5 & 6 = 60
- L.C.M. of 5, 6 & 7 = 210



Example: 8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ?

- H.C.F. of 8 and 4 = 4
- H.C.F. of 7 and 8 = 1
- H.C.F. of 3 and 9 = 3

H.C.F. of 2 and 1 = 1

Numbers Followed by Their Product



Example: 1, 3, 3, 9, 27, 243, ?

- $1 = 1$
- $1 \times 3 = 3$
- $3 \times 3 = 9$
- $3 \times 9 = 27$
- $9 \times 27 = 243$
- Next Number = Product of Previous Two Numbers
- Next Number = $27 \times 243 = 6561$

Use of Digit Sum



Example: 11, 13, 17, 25, 32, ...?

- $13 = 11 + (1 + 1)$
- $17 = 13 + (1 + 3)$

- $25 = 17 + (1 + 7)$
- $32 = 25 + (2 + 5)$
- Next Number = Previous Number + Sum of the digits of previous number
- Next Number = $32 + (3 + 2) = 37$

Different Types of Question Asked in Exams

1. Finding Wrong Term – Series

It is a series in which all others except one are similar in some respect. All terms follow the same pattern except one. That is wrong term.



Example: 5, 10, 17, 24, 37, 50, 65.

- $5 = 2^2 + 1$, $10 = 3^2 + 1$, $17 = 4^2 + 1$, $26 = 5^2 + 1$, $37 = 6^2 + 1$, $50 = 7^2 + 1$, $65 = 8^2 + 1$

2. Finding Missing Term – Series

It is a series is given in which a blank space or question mark is provided in place of any one term of the series. The term at the blank space follows the same pattern as followed by other terms. We are required to find the missing term to replace the blank space or question mark



Example: 49, 56, 64, 72, ?, 90, 100

- $49 = 7^2$; $56 = 7^2 + 7$; $64 = 8^2$; $72 = 8^2 + 8$; **$81 = 9^2$** ; $90 = 9^2 + 9$; $100 = 10^2$
- $T_5 = 81 = 9^2$

3.4 Alpha-Numeric Series

These series involve the use of both the letters of the alphabet as well as the numbers. It is a two-line series. One line is a number series while the other line is an alphabet series. The terms of both the series follow the same pattern/rule. One of these two series is completely known. We must find the required number of the incomplete series Letters or Numbers.



Example:

2, 7, 17, 37, 77

3, a, b, c, d

- $7 = 2 \times 2 + 3$; $17 = 7 \times 2 + 3$; $37 = 17 \times 2 + 3$; $77 = 37 \times 2 + 3$

Similarly,

- $a = 3 \times 2 + 3 = 9$; $b = 9 \times 2 + 3 = 21$; $c = 21 \times 2 + 3 = 45$; $d = 45 \times 2 + 3 = 93$

3.5 Analogy of Number Series

Analogy

- The meaning of analogy is 'similar properties or similarity.'
- If an object or word or digit or activity shows any similarity with another object or word or digit or activity in terms of properties, type, shape, size, trait etc., then the particular similarity will be called analogy.
- For example, cricket: ground and chess: table are the analogous pairs (why?).

- In fact, both pairs of words have similar relationship in terms of place of playing as cricket is played in the ground and similarly chess is played on the table.

Types of Analogy Reasoning

- Letter/Word Based Analogy
- Number Based/Numerical Analogy
- Letter and Number Based Analogy (Mixed Analogy)
- Image Based Analogy
- General Knowledge Based Analogy

Number Based/Numerical Analogy

- In this type of analogy reasoning, a pair of numbers are given in a certain similarity between them.
- Another number is also given with the same similarities.
- We have to find out the similarities between the first two numbers and choose the number from the given alternatives, which carries the same similarities to the "?", as the first two carry.
- Various types of number-based analogy are Addition/ Subtraction/ Multiplication with the constant numbers, Square/Cube of the constant numbers, Operation of Sum of the digits, and so on.



Example: Find the missing term in given series analogy

$$3 : 12 :: 5 : ?$$

Solution:

$$3 : 12 = (3^2) + 3$$

Similarly

$$5 : (5^2) + 5 = 30$$



Example: $14 : 9 :: 26 : ?$

$$14 = (2 \times 9 - 4)$$

Similarly

$$26 = (2 \times 15 - 4)$$

$$? = 15$$

3.6 Number Classification

- In number classification reasoning questions, a group of letters and numbers is given.
- On behalf of numeric values and their position numbers follow mathematical operation/rules, hence form a group.
- Candidates are required to select the option which does not belong to that same group.



Example: Choose the number which is DIFFERENT from the rest?

21, 39, 51, 83

Solution:

83 is the only prime number in the group. Therefore 83 is odd one in the given group of numbers.



Example: Choose the number which is DIFFERENT from the rest?

145, 197, 257, 399

Solution:

Each of the numbers except 399, is one more than the square of a certain number.

3.7 Problems Based on Number Series, Analogy and Classifications



Example: 1

Insert the missing number in the following series:

5, 8, 12, 17, 23,, 38

Solution:

$$T_2 - T_1 = 8 - 5 = 3$$

$$T_3 - T_2 = 12 - 8 = 4$$

$$T_4 - T_3 = 17 - 12 = 5$$

$$T_5 - T_4 = 23 - 17 = 6$$

$$T_6 - T_5 = X - 23 = 7$$

$$T_7 - T_6 = 38 - X = 8$$

- $X = 38 - 8 = 7 + 23 = 30$



Example: 2

Insert the missing number in the given series:

4, 9, 20, 43, 90, ____

Solution:

$$T_2 = 9 = 2 \times 4 + 1$$

$$T_3 = 20 = 2 \times 9 + 2$$

$$T_4 = 43 = 2 \times 20 + 3$$

$$T_5 = 90 = 2 \times 43 + 4$$

$$T_6 = 2 \times 90 + 5 = 185$$

4, 9, 20, 43, 90, 185



Example: 3

Insert the missing number in the given series:

1, 1, 4, 8, 9, 27, 16, ____

Solution:

First Series - 1, 4, 9, 16,

or $1^2, 2^2, 3^2, 4^2, \dots \dots \dots$

Second Series - 1, 8, 27, ...

Or $1^3, 2^3, 3^3, \dots$

Or

$1^3, 2^3, 3^3, 4^3, \dots$

Missing Number = 64



Example: 4

Fill in the missing number in the following series:

11, 10, ?, 100, 1001, 1000, 10001,

Solution:

$$T_2 = 10, T_4 = 100, T_6 = 1000, \dots$$

$$T_1 = 11, T_3 = ?, T_5 = 1001, T_7 = 10001, \dots$$

Missing term, $T_3 = 101$

11, 10, 101, 100, 1001, 1000, 10001,



Example: 5

Find the fifth term in the following series:

99, 95, 86, 70,

Solution:

$$T_2 - T_1 = 95 - 99 = -4 = -2^2$$

$$T_3 - T_2 = 86 - 95 = -9 = -3^2$$

$$T_4 - T_3 = 70 - 86 = -16 = -4^2$$

$$T_5 - T_4 = X - 70 = -25 = -5^2$$

$$T_5 = X = 70 - 25 = 45$$

99, 95, 86, 70, 45



Example: 6

Find the number corresponding to question mark in the following series:

0, 3, 12, 30, ?, 105, 168

Solution:

$$T_2 - T_1 = 3 - 0 = 3$$

$$T_3 - T_2 = 12 - 3 = 9 = 3 + 6$$

$$T_4 - T_3 = 30 - 12 = 18 = 9 + 9$$

$$T_5 - T_4 = X - 30 = ? = 18 + 12 = 30$$

$$T_6 - T_5 = 105 - X = ? = 30 + 15 = 45$$

$$T_7 - T_6 = 168 - 105 = 63 = 45 + 18$$

0, 3, 12, 30, X, 105, 168

Missing term, $X = 30 + 30 = 105 - 45 = 60$



Example: 7

Find out the wrong number in the following series:

455, 445, 465, 435, 485, 415, and 475

Solution:

First series,

455, 465, 485, and 475

$465 - 455 = 10$; $485 - 465 = 20$; and $475 - 485 = -10$ (Difference should be $20 + 10 = 30$)

Second series,

445, 435, and 415

$435 - 445 = -10$; and $415 - 435 = -20$

In first series numbers should follow a difference of +10, +20, +30

Therefore 475 is wrong

Correct number = $485 + 30 = 515$



Example: 8

Find out the wrong number in the following series:

1, 5, 11, 19, 29, and 55

Solution:

$5 - 1 = 4$

$11 - 5 = 6$

$19 - 11 = 8$

$29 - 19 = 10$

$55 - 29 = 26$ (Difference should be $10 + 2 = 12$)

Therefore 55 is wrong

Correct number = $29 + 12 = 41$



Example: 9

Find out the wrong number in the following series:

2, 4, 4, 16, 8, 256, and 64

Solution:

First Series

2, 4, 8, and 64

$2^1, 2^2, 2^3$, and 2^6 (2^6 should be replaced by 2^4)

Second series

4, 16, and 256 or $2^2, 2^4$, and 2^8



Example: 10

In the following questions a number series is given. After the series a number is given followed by a, b, c, d and e. Complete the series starting with the number given following the sequence of the given series.

1, 9, 65, 393

2, a, b, c, d, e

Solution:

$$9 = 1 \times 8 + 1$$

$$65 = 9 \times 7 + 2$$

$$393 = 65 \times 6 + 3$$

1, 9, 65, 393

2, a, b, c, d, e

$$9 = 1 \times 8 + 1; 65 = 9 \times 7 + 2; 393 = 65 \times 6 + 3$$

$$a = 2 \times 8 + 1 = 17$$

$$b = 17 \times 7 + 2 = 121$$

$$c = 121 \times 6 + 3 = 729$$

$$d = 729 \times 5 + 4 = 3649$$

$$e = 3649 \times 4 + 5 = 14601$$



Example: 11

Find out the wrong number in the following series:

3, 10, 24, 54, 108, 220, and 444

Solution:

$$10 = 2 \times 3 + 4$$

$$24 = 2 \times 10 + 4$$

$$54 = 2 \times 24 + 6$$

$$108 = 2 \times 54 + 0$$

$$220 = 2 \times 108 + 4$$

$$444 = 2 \times 220 + 4$$

Correct number $52 = 2 \times 24 + 4$ and $108 = 2 \times 52 + 4$



Example: 12 Find the missing term in given number series analogy.

11 : 17 :: 19 : ?

Solution:

11 : 17 alternate prime number (skipping 13)

Similarly

19 : 29 alternate prime number (skipping 23)



Example: 13 Find the missing term in given number series analogy.

Analytical Skills

$$3 : 27 :: 4 : ?$$

Solution:

$$\text{Second term} = (\text{First term})^3$$

$$\text{Fourth term} = (\text{Third term})^3$$



Example: 14 Find the missing term in given number series analogy.

$$12 : 30 :: 20 : ?$$

Solution:

$$12 = 3^2 + 3; 30 = 5^2 + 5 :: 20 = 4^2 + 4; ? = 6^2 + 6$$

Therefore,

$$? = 42$$



Example: 15 Find the missing term in given number series analogy.

$$3 : 10 :: 8 : ?$$

Solution:

$$3 = 2^2 - 1,$$

$$10 = 3^2 + 1,$$

$$8 = 3^2 - 1,$$

$$? = 4^2 + 1 = 17$$



Example: 16 Find the missing term in given number series analogy.

$$48 : 122 :: 168 : ?$$

Solution:

$$48 = 7^2 - 1; 122 = 11^2 + 1 :: 168 = 13^2 - 1; ? = 17^2 + 1 = 290$$



Example: 17 Find the missing term in given number series analogy.

$$5 : 21 :: 7 : ?$$

Solution:

$$5 \times 5 = 25 - 4 = 21 ; 7 \times 7 = 49 - 6 = 43$$



Example: 18 Find the missing term in given number series analogy.

$$182 : ? :: 210 : 380$$

Solution:

$$210 = 15^2 - 15 : 380 = 20^2 - 20, (15+5=20)$$

Similarly

$$182 = 14^2 - 14 : 19^2 - 19 = 342, (14+5=19)$$



Example: 19 Find the missing term in given number series analogy.

49: 21 :: 169:?

Solution:

$$7^2 : 7 \times 3 :: 13^2 : 13 \times 3$$

Or, 49 : 21 :: 169 : 39



Example: 20 Find the missing term in given number series analogy.

2149370: 2854610: 2735630: ?

Solution:

$$2+1+4+9+3+7+0=26$$

$$2+8+5+4+6+1+0=26$$

$$2+7+3+5+6+3+0=26$$

$$\text{Therefore, } 9+4+3+7+1+0+2=26$$



Example: 21 Find the missing term in given number series analogy.

Solution:

7: 56 :: 9 : ?

The relationship is $x : x(x+1)$

Therefore 7: 7×8 :: 9 : 9×10



Example: 22 Find the missing term in given number series analogy.

Solution:

20 : 50 :: 100 : ?

$$50 = 20 \times 2 + 20/2,$$

$$\text{So, } 100 \times 2 + 100/2 = 250$$



Example: 23 Find the missing term in given number series analogy.

8:256 :: ?

Solution:

$$8 \times 8 = 64 \times 4 = 256,$$

$$10 \times 10 = 100 \times 5 = 500$$



Example: 24 Choose the number which is DIFFERENT from the rest?

A. 324 B. 244 C. 136 D. 352

Solution:

Sum of the digits of each of the numbers is 10 except 324.



Example: 25 Choose the number which is DIFFERENT from the rest?

A. 15 B. 63 C. 143 D. 257

Solution:

Each of the numbers except 257, is one less than the square of a certain number.



Example: 26 Choose the number which is DIFFERENT from the rest?

A. 35 B. 49 C. 50 D. 63

Solution:

Each of the numbers except 50, is divisible by 7.



Example: 27 Choose the number which is DIFFERENT from the rest?

A. 385 B. 572 C. 671 D. 427

Solution:

In all numbers except 427, the middle digit is the sum of the other two.



Example: 28 Choose the number which is DIFFERENT from the rest?

A. 2384 B. 1592 C. 3756 D. 3629

Solution:

In all numbers except 3629, the last digit is two times the first.



Example: 29 Choose the number which is DIFFERENT from the rest?

A. 120 B. 168 C. 290 D. 380

Solution:

Each of the numbers except 380, is either one less or one more than the square of a certain number.



Example: 30 Choose the number which is DIFFERENT from the rest?

A. 3759 B. 2936 C. 6927 D. 4836

Solution:

In all numbers except 2936, the sum of second and last digits is twice the sum of first and third digits.



Example: 31 Which Pair does NOT belong with the others?

A. 16 – 64 B. 9 – 36 C. 36 – 216 D. 49 – 343

Solution:

All pairs except 9-36 contain square and cube of the same number. e.g., $36 = 6^2$ and $216 = 6^3$.



Example: 32 Which Pair does NOT belong with the others?

A. 6 – 15 B. 21 – 43 C. 25 – 51 D. 29 – 59

Solution:

In all pairs except 6-15, 2nd number = (1st number \times 2) + 1.



Example: 33 Which Pair does NOT belong with the others?

- A. 4 : 16 B. 6 : 42 C. 8 : 24 D. 17 : 54

Solution:

In all pairs except 17 : 54, second number is a multiple of the first.



Example: 34 Which Pair does NOT belong with the others?

- A. 21 : 6 B. 28 : 4 C. 42 : 12 D. 84 : 24

Solution:

In all pairs except 28 : 4, 1st number = 2nd number \times 3.5.

Summary

The key concepts learned from this unit are: -

- Understood what is series?
- We have discussed about different type of series.
- Understood what is analogy?
- We solved problems based on number analogy.
- Understood what is number classification,
- We solved problems based on number series, Analogy and number classification.

Keywords

- Sequence is a set of numbers, follows a definite order/rule. Series is a sequence of numbers. Arithmetic series is one in which the difference between any two consecutive terms is always the same. Geometric series is one in which the ratio of any two consecutive terms is always the same.

Self Assessment

1. Insert the missing number 5, 8, 12, 17, 23, , 38
A. 29
B. 30
C. 32
D. 25
2. Fill in the missing number 5, 18, 10, 12, 15, ?
A. 4
B. 8
C. 6
D. 1
3. Fill in the missing number 2, 6, 3, 4, 20, 5, 6, ?, 7
A. 25

- B. 42
- C. 24
- D. 18

4. In the following number series a wrong number is given. Find out the wrong number.

455, 445, 465, 435, 485, 415, 475

- A. 475
- B. 465
- C. 435
- D. 455

5. Which of the following numbers will come in place of (C)?

1 9 65 393

2 (A) (B) (C) (D) (E)

- A. 490
- B. 729
- C. 854
- D. 73

6. Find the Wrong number in the following number series.

3, 7, 16, 35, 70, 153

- A. 70
- B. 16
- C. 153
- D. 35

7. What will come in place of question mark (?) in the given number series?

123, 140, 106, 157, 89, ?

- A. 174
- B. 139
- C. 198
- D. 169

8. Find out the wrong number in the sequence: 40960, 10240, 2560, 640, 200, 40, 10

- A. 2560
- B. 200
- C. 640
- D. 40

9. Find the value of x in the series 2, 6, 30, 210, x, 30030, ...

- A. 2310

- B. 1890
- C. 2520
- D. 2730

10. Insert the missing number 3, 18, 12, 72, 66, 396, ?

- A. 300
- B. 380
- C. 350
- D. 390

11. In the following number series only one number is wrong. Find out the wrong number.

7, 12, 40, 222, 1742, 17390, 208608

- A. 7
- B. 12
- C. 40
- D. 1742

12. In the following number series only one number is wrong. Find out the wrong number.

6, 91, 584, 2935, 11756, 35277, 70558

- A. 91
- B. 70558
- C. 584
- D. 2935

13. Find out the missing term in the following number series analogy.

11. 16: 56:: 32;?

- A. 96
- B. 112
- C. 120
- D. 128

14. Find out the missing term in the following number series analogy.

23: 13 :: 54:?

- A. 45
- B. 39
- C. 40
- D. 41

15. Find out the missing term in the following number series analogy.

10 : 91 :: 9 : ?

- A. 69
- B. 72
- C. 89

D. 97

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. B | 4. A | 5. B |
| 6. A | 7. A | 8. B | 9. A | 10. D |
| 11. D | 12. C | 13. B | 14. D | 15. B |

Review Questions

In each question below (1-3), a number series is given in which one number is wrong. Find out the wrong number

1. 484, 240, 120, 57, 26.5, 11.25, 3.625
2. 5, 7, 16, 57, 244, 1245, 7506
3. 6, 7, 16, 41, 90, 154, 292
4. 201, 202, 204, 207, ... What number should come next?
5. Find the missing series 3, 12, 27, 48, 75, 108, ?
6. Find the missing series 2, 3, 8, 63, ?
7. Find the missing series 1, 7, 3, 9, 6, 12, 10, 16, 15, ?
8. Find the next two terms: 17, 32, 19, 29, 21, 26, 23, ?, ?
9. 2, 1, (1/2), (1/4), ... What number should come next?
10. 10. 544, 509, 474, 439, ... What number should come next?
11. 11. 29282, 2662, 242, 22, ? What number should come next?
12. Find the next analogy term 12: 20::?
13. Which Pair does NOT belong with the others?
A. 14, 9 B. 17, 8 C. 21, 6 D. 42, 3

**Further Readings**

- Quantitative Aptitude for Competitive Examinations, By Dr. R S Aggarwal, S Chand Publishing
- A Modern Approach, To Verbal & Non-Verbal Reasoning, By Dr. R S Aggarwal, S Chand Publishing
- Magical Book on Quicker Maths, By M Tyra, Banking Service Chronicle
- 4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Unit 04: Alphabet Series

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Summary

Keywords

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- What is Alphabet series?
- What is alphanumeric series?
- Identify the pattern in the given series.
- Solve different problems based on alphabet and alphanumeric series completion.
- What is alphabet analogy?
- Solve problems based on Analogy and Classification of alphabet series.

Introduction

In almost all competitive exams, problems related to alphabet series are asked. Letter and Symbol Series are a sequential order of letters, numbers or both arranged such a way that each term in the series is obtained according to some specific rules. These rules can be based on mathematical operations, place of letters in alphabetical order etc.

4.1 Alphabetical/ Letter Series

- In this type of questions, some alphabetical letters are given.
- They all form a series and the series changes in certain order.
- The series may also have one or more letters missing.
- The students are required to observe that specific order in which the series changes and then complete the series.

- Similarly, the students have to decide about the missing letter that would suit for the blank space if they continue to change in some order.

Alphabets of English Literature

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

- We all know there is 26 alphabets in English literature, in which the word **EJOTY** is very helpful in finding the position in letter series.

E	J	O	T	Y
5	10	15	20	25

- For example, if we want to find the position of the alphabet 'S', then as we know that 'T' is 20, so 'S' is $20 - 1 = 19$. Also, we can find the position of an alphabet from the end by subtracting its value from 27. For example, the position of D from the end is $27 - 4 = 23$.
- The pattern below is very useful to find many types of question in Alphabet Series when we write the last 13 alphabets in front of the first 13 alphabets of the English:

Ranking	1	2	3	4	5	6	7	8	9	10	11	12	13
Forward Ranking of Letters	A	B	C	D	E	F	G	H	I	J	K	L	M
Backward Ranking of Letters	Z	Y	X	W	V	U	T	S	R	Q	P	O	N

Backward ranking = $27 - \text{Forward ranking}$

Forward ranking = $27 - \text{Backward ranking}$



For Example: Find the letter in the 9th position when the alphabets are written in reverse alphabetical order.

Solution:

BR-9 (Backward ranking)

Forward Ranking = $27 - \text{BR} = 27 - 9 = 18 = \mathbf{R}$

Finding missing terms in Alphabet/Letter Series



Example: Find the Missing term-

a, d, c, f, ?, h, g, ?, i

Solution:

In the present question two separate series are these

Series 1 :- a, c, ?, g, i

Series 2:- d, f, h, ?

Both series is with difference of +2., So, terms are e, j.



Example: Find the Missing term-

Z X Y W V T U S R P Q O?

Solution:

The series is fragmented in to group of 4 letters line.

WXYZ → written as ZXYW

STUV → VTUS

OPQR → RPQO

KLMN → NLMK

so next term will be NL.

4.2 Alphanumeric Series

- Letter + number
- A4A, C9D, G20I, O43P, E90Y, K185J



Example: In the given series find the term which is wrong.

G4T, J10R, M20P, P43N, S90L

Solution:

The number figure in each term is following a logic ($X^2 + 1$)

Hence it should be 9 in place of 10.

4.3 Alphabetical Pattern Series

Repetitive factor/pattern

ab _ _ b _ bba _ _

abb/aab/abb/aab



Example: Find the missing series.

xy _ kx _ zk _ yzk xyz _ _

Solution:

The series is

xyzk/xyzk/xyzk/xyzk/x
=> zyxx



Example: In the following letter series, some of the letters are missing, which are given in that order as one of the alternatives below it. Choose the correct alternatives:
bc_b_c_b_ccb

Solution:

The series is:

bccb/bccb/bccb.

4.4 Analogy of letter Series

- In this type of analogy reasoning, a pair of letters are given in a certain similarity between them.
- Another letter is also given with the same similarities.
- We have to find out the similarities between the first two letters and choose the letter from the given alternatives, which carries the same similarities to the "?", as the first two carry.



Example: DRIVEN: EIDRVN: BEGUM: ?

Solution:

Fifth and third letters of the first term are first and second letters of the second term and first two letters of the first term are third and fourth letters of the second term. Therefore, missing term is MGBEU



Example: NUMBER: UNBMER::GHOST:?

Solution:

First two letters of the first term are in reverse order in the second term and third and fourth letter are also in reverse order after that no change in position. Therefore, missing term is HGSOT.

4.5 Alphabet Classification

- In letter series classification reasoning questions, usually four group of letters are given.
- Three of them are similar to each other in some manner.
- While one is different and we have to choose different one.



Example: Select the one which is different from the other three responses.

A. FIJL B. RUVX C. DGHJ D. NPQS

Solution:

Except the fourth option, in all other options the differences between letters are same.

1st option: F - (+3) → I - (+1) → J - (+2) → L

2nd option: R - (+3) → U - (+1) → V - (+2) → X

3rd option: D - (+3) → G - (+1) → H - (+2) → J

But, 4th option: N - (+2) → P - (+1) → Q - (+2) → S

Therefore, required answer is option D.



Example: Select the one which is different from the other three responses.

A. BADC B. XWZY C. VUST D. NMPO

Solution:

Except the third option, in all other options the differences between letters are same.

1st option: $B - (-1) \rightarrow A - (+3) \rightarrow D - (-1) \rightarrow C$

2nd option: $X - (-1) \rightarrow W - (+3) \rightarrow Z - (-1) \rightarrow Y$

4th option: $N - (-1) \rightarrow M - (+3) \rightarrow P - (-1) \rightarrow O$

But, 3rd option: $V - (-1) \rightarrow U - (-2) \rightarrow S - (+1) \rightarrow T$

Therefore, required answer is option C.

4.6 Problems based on letter Series, Analogy and Classifications



Example: 1 What should come in the place of question mark (?) in the following letter series based on the English alphabetical order?

BE, GJ, LO, QT, ?

Solution:

The given series of the pattern +3 and +2, Therefore $Q(17) + 3 = T(20) + 2 = V(22) + 3 = Y(25)$.

So, missing term is VY.



Example: 2 Which set of letters when sequentially placed at the gaps in the given letter series shall complete it?

qr_srq_p_rs_r_p

Solution: The correct sequence is **pqrs/srqp/pqrs/srqp**.

And the pattern is pqrs and srqp.

Therefore, Missing letters are pspqsq.

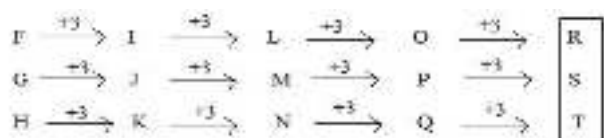


Example: 3

Find the next term in the given series:

FGH, IJK, LMN, OPQ, ?

Solution:

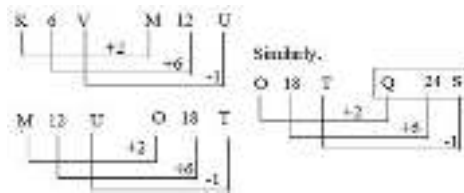


Example: 4

Find the next term in the given alphanumeric series:

K6V, M1

Solution: 2U, O18T, ?



Example: 5

F2, __, D8, C16, B32 Which letter/number should complete the series?

Solution:

The letters decrease by 1; the numbers are multiplied by 2. Therefore, missing letter/number is E4.

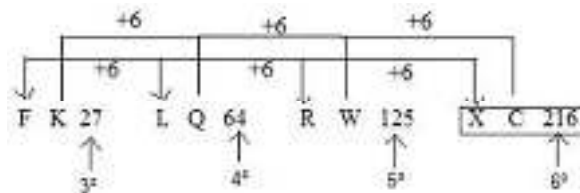


Example: 6

Find the number/letter corresponding to question mark in the following series:

FK27, LQ64, RW125, ?

Solution:



Example: 7

Find out the missing term in the following letter series analogy:

PAPER: SCTGW:: MOTHER : ?

Solution:

+3 P ↓ S, +2 A ↓ C, +4 P ↓ T, +2 E ↓ G, +5 R ↓ W

Similarly, +3 M ↓ P, +2 O ↓ Q, +4 T ↓ X, +2 H ↓ J, +5 E ↓ J, +2 R ↓ T

Therefore, missing analogy is PQXJJT.



Example: 8

Find out the missing term in the following letter series analogy:

POPULAR: QPQVMBS::? : GBNPVT

Solution:

+1 P ↓ Q, +1 O ↓ P, +1 P ↓ Q, +1 U ↓ V, +1 L ↓ M, +1 A ↓ B, +1 R ↓ S

Similarly, -1 G ↓ F, -1 B ↓ A, -1 N ↓ M, -1 P ↓ O, -1 V ↓ U, -1 T ↓ S

Therefore, missing analogy is FAMOUS.



Example: 9

Find out the missing term in the following letter series analogy:

CALCUTTA: GEPGYXXE :: ? : FSQFCE

Solution:

+4 C ↓ G, +4 A ↓ E, +4 L ↓ P, +4 C ↓ G, +4 U ↓ Y, +4 T ↓ X, +4 T ↓ X, +4 A ↓ E

Similarly, -4 F ↓ B, -4 S ↓ O, -4 Q ↓ M, -4 F ↓ B, -4 C ↓ Y, -4 E ↓ A

Therefore, missing analogy is BOMBYA.



Example: 10

Find out the missing term in the following letter series analogy:

PRLN: XZTV:: JLFH : ?

Solution:

P +8 X, R +8 Z, L +8 T, N +8 V

Similarly, J +8 R, L +8 T, F +8 N, H +8 P

Therefore, missing analogy is RTNP.



Example: 11

Find out the missing term in the following letter series analogy:

ACFJ: OUZJ:: SUXB : ?

Solution:

As, Similarly,

A + 14 O :: S +14 G, C +18 U :: U+18 M, F +20 Z :: X +20 R, J +0 J :: B +0 B

Therefore, missing analogy is GMRB.



Example: 12

Find out the missing term in the following letter series analogy:

ACE: HIL:: MOQ : ?

Solution:

As, Similarly, A +7 H :: M +7 T, C +6 I :: O +6 U, E +7 L :: Q +7 X

Therefore, missing analogy is TUX.



Example: 13

Find out the missing term in the following letter series analogy:

ACBD : EFGH :: OQPR ?

Solution:

A +2 C -1 B +2 D +1 : E +1 F +1 G +1 H

Similarly, O +2 Q -1 P +2 R +1 : S +1 T +1 U +1 V.

Therefore, missing analogy is STUV.



Example: 14

Analytical Skills

Find out the missing term in the following letter series analogy:

TSR: FED:: WVU ?

Solution:

The letters are consecutive and written in reverse order. R-14 D, Similarly, U-14 G.

Therefore, missing analogy is IHG.



Example: 15

Find out the missing term in the following letter series analogy:

BCDA : STUR :: KLMJ : ?

Solution:

In each group the first three letters are consecutive and they follow the fourth letter.

BCDA (+17 in each term) : STUR :: KLMJ (+17 in each term) : BCDA

Therefore, missing analogy is BCDA.



Example: 16

Find the missing term in given letter series analogy.

CEG:EGC ::LNP:?

Solution:

The second set EGC is formed by simply putting the first letter of CEG at last to form EGC.

Therefore, missing analogy is NPL.



Example: 17

Find the missing term in given letter series analogy.

KLM:PON::NOP:?

Solution:

Because KLM having forward rankings are 11, 12 & 13 from A to Z, this corresponds to PON, which are having backward ranking 11, 12 and 13 from Z to A in reverse order. Hence NOP having FRs are 14, 15, 16, which will correspond to MLK having backward rankings 14, 15 and 16.

Therefore, missing analogy is MLK.



Example: 18

Find the missing term in given letter series analogy.

ACE: FGH:: LNP : ?

Solution:

The three letters moved 5, 4 and 3 and steps forward respectively.

Therefore, missing analogy is QRS (L+5 Q, N+4 R, P+3 S)



Example: 19

Find the missing term in given letter series analogy.

ACFJ: ZXUQ:: EG IN : ?

Solution:

As, $A \downarrow Z$, $C \downarrow X$, $F \downarrow U$, $J \downarrow Q$ (Opposite letters)

similarly, $E \downarrow V$, $G \downarrow T$, $I \downarrow R$, $N \downarrow M$ (Opposite letters)

Therefore, missing analogy is VTRM



Example: 20

Find the missing term in given letter series analogy.

SINGER: QGLECP:: MONSTER: ?

Solution:

$S - 2 Q$, $I - 2 G$, $N - 2 L$, $G - 2 E$, $E - 2 C$, and $R - 2 P$

Similarly $M - 2 K$, $O - 2 M$, $N - 2 L$, $S - 2 Q$, $T - 2 R$, $E - 2 C$, $R - 2 P$

Therefore, missing analogy is KMLQRCP



Example: 21. Select the one which is different from the other three responses.

A. DWHS B. BYDW C. CWFS D. EVJQ

Solution:

Except the third option, all other options are the Pairs of opposite letters

$D \leftrightarrow W$; $H \leftrightarrow B$ $Y \leftrightarrow S$; $D \leftrightarrow W$; $E \leftrightarrow V$; $J \leftrightarrow Q$

But in third option the opposite letter of C is X and that of F is U.



Example: 22. Select the one which is different from the other three responses.

A. KNM B. JMK C. CFD D. GJH

Solution:

Except the first option, in all other options the differences between letters are same.

2nd option: $J - (+3) \rightarrow M - (-2) \rightarrow K$

3rd option: $C - (+3) \rightarrow F - (-2) \rightarrow D$

4th option: $G - (+3) \rightarrow J - ((-2) \rightarrow H$

But, 1st option: $K - (+3) \rightarrow N - (-1) \rightarrow M$

Therefore, required answer is A.



Example: 23 Select the one which is different from the other three responses.

A. JKL B. FGI C. MNR D. TUW

Solution:

Except the first option, in all other options the differences between letters are same.

2nd option: $F - (+1) \rightarrow G - (+2) \rightarrow I$

3rd option: $M - (+1) \rightarrow N - (+2) \rightarrow R$

4th option: $T - (+1) \rightarrow U - (+2) \rightarrow W$

But, 1st option: $J - (+1) \rightarrow K - (+1) \rightarrow L$



Example: 24. Select the one which is different from the other three responses.

A. DHLP B. TXBF C. JNRV D. YBEH

Solution:

Except the fourth option, in all other options the differences between letters are same.

1st option: D – (+4) → H – (+4) → L – (+4) → P

2nd option: T – (+4) → X – (+4) → B – (+4) → F

3rd option: J – (+4) → N – (+4) → R – (+4) → V

But, 4th option: Y – (+3) → B – (+3) → E – (+3) → H



Example: 25. Select the one which is different from the other three responses.

A. IAME B. VNZR C. KCOG D. FKOR

Solution:

Except the fourth option, in all other options the differences between letters are same.

1st option: I – (-8) → A – (+12) → M – (-8) → E

2nd option: V – (-8) → N – (+12) → Z – (-8) → R

3rd option: K – (-8) → C – (+12) → O – (-8) → G

But, 4th option: F – (+5) → K – (+4) → O – (+3) → R



Example: 26. Select the one which is different from the other three responses.

A. URT B. GDF C. GDF D. NKM

Solution:

Except the fourth option, in all other options the difference between letters is same.

1st option: U – (-3) → R – (+2) → T

2nd option: G – (-3) → D – (+2) → F

3rd option: P – (-3) → L – (+2) → N

But, 4th option: N – (-4) → K – (+2) → M

Summary

The key concepts learned from this unit are: -

- Understood what is alphabet series?
- We have discussed about different types of questions of alphabet series.
- Understood what is letter analogy?
- Understood what is alphabet classification,
- We solved problems based on alphabet series, Analogy and alphabet classification.

Keywords

- Alphabet Series; Forward Ranking of Letters; Backward Ranking of Letters, Letter Analogy, Letter Classification;

Self Assessment

1. In given series one term is missing. Choose the correct alternative from the given options that will complete the series.

AZWD, CXUF, ?, GTQJ

- A. EVSH
- B. EUTH
- C. EUSH
- D. EVIP

2. Which letter is exactly in the middle of 20th letter from the right and 11th letter from the left?

- A. I
- B. J
- C. H
- D. O

3. Find the missing letter/number in the given series

N5V, K7T, ?, E14P, B19N

- A. H9R
- B. H10Q
- C. H10R
- D. I10R

4. In following alphabet series, one term missing as shown by question mark (?). Choose missing term from options.

U, O, I, ?, A

- A. E
- B. C
- C. S
- D. G

5. In following alphabet series, two terms are missing as shown by question mark. Choose missing term from options.

Y, W, U, S, Q, ?, ?

- A. N,J
- B. M,L
- C. J,R
- D. O,M

6. Find the missing term.

WFB, TGD, QHG, ?

- A. NIJ
- B. NIK
- C. NJK
- D. OIK

7. ELFA, GLHA, ILJA, _____, MLNA

- A. OLPA
- B. KLMA
- C. LLMA
- D. KLLA

8. Complete the series.

ba_ba_bac_acb_cbac

- A. aacb
- B. bbca
- C. ccba
- D. cbac

9. n_mnp_ _ p_ npmn_ mnp

- A. pmnpm
- B. pmnpp
- C. pmnmp
- D. pnpmn

10. Find out the missing term in the following letter series analogy.

MASTER: OCUVGT:: LABOUR:?

- A. NCDQWT
- B. NDERWT
- C. NBCRWT
- D. NEDRWT

11. Find out the missing term in the following letter series analogy.

RIDE: LNBE:: HELP : ?

- A. NINP
- B. BAJP
- C. JPCH
- D. BJJP

12. Find out the missing term in the following letter series analogy.

MUMBAI: LTLAZH : : DELHI: ?

- A. CDKGJ
- B. IHLED
- C. CDKGH
- D. BCKGH

13. Find out the missing term in the following letter series analogy.

RATIONAL: RATNIOLA::_____: TRILBA

- A. TIRLAB
- B. TRIBAL
- C. TRIALB
- D. TIRBAL

14. Find out the missing term in the following letter series analogy.

HEATER: KBDOHO:: COOLER:?

- A. ALRHV
- B. FLRIHO
- C. FLIRHO
- D. FRLIHO

15. Select the one which is different from the other three responses.

- A. AIUE
- B. PIRE
- C. VALT
- D. TORE

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. A | 2. A | 3. C | 4. A | 5. D |
| 6. B | 7. D | 8. C | 9. C | 10. A |
| 11. D | 12. C | 13. B | 14. B | 15. A |

Review Questions

1. Find out the missing term in the following letter series analogy.
ACEG: DFHJ:: QSUW :?
2. Find out the missing term in the following letter series analogy.
EGIK: FILO:: FHJL : ?
3. Find out the missing term in the following letter series analogy.
AROUND: RAUODN::GROUND:?
4. Find out the missing term in the following letter series analogy.

APPROACHED: ROACHEDAPP:: BARGAINED:?

5. Find out the missing term in the following letter series analogy.
AFKP: DINS:: WBGL : ?
6. Find the odd one out from the given alternatives:
A. NOP B. KLM C. UVW D. HIJ
7. Arrange these letters of each group to make a meaningful word and then find the odd one out.
A. ORFU B. VIDEID C. GHIET D. VEENS
8. Identify which one of the given alternatives will be another member of the group of that class.
Lucknow, Patna, Bhopal, Jaipur, ?
A. Pune B. Indore C. Shimla D. Mysore
9. Four of the following five are alike in a certain way and so form a group. Which is the one that does not belong to that group?
A. Water B. Juice C. Petrol D. Sugar
10. Select the one which is different from the other three responses.
A. Mayor B. Lawyer C. Governor D. Legislator



Further Readings

- Quantitative Aptitude for Competitive Examinations, By Dr. R S Aggarwal, S Chand Publishing
- A Modern Approach, To Verbal & Non-Verbal Reasoning, By Dr. R S Aggarwal, S Chand Publishing
- Magical Book on Quicker Maths, By M Tyra, Banking Service Chronicle
- 4. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Unit 05: Coding-Decoding

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5.6 Mixed Number Coding/ Deciphering Message Number and Symbol Codes

Summary

Keywords

Self Assessment

Answers for Self Assessment

Review Questions

Further Reading

Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Different types of coding problems of reasoning.
- Solve problems related to Letter and Direct Coding and Decoding.
- Solve problems related to number and Symbol coding.
- What is deciphering message in coding and decoding.
- Solve problems related to deciphering message word codes.
- Solve problems related to deciphering message number and symbol codes.

Introduction

Coding-Decoding is a process of encrypt or decrypt any word, letter, or sentence in a pattern or code based on some sets of rules. Coding and Decoding of an information done with some rules or patterns. Coding-Decoding helps candidates to improve their logical reasoning skills and ability to focus. Coding is a process used to encrypt a word, a number in a particular code or pattern based on some set of rules. Decoding is a process to decrypt the pattern into its original form from the given codes. Coding-Decoding Questions are used to judge the ability to decipher the rule or pattern, which is applied at the time of coding the information. To solve the Questions based on Coding-Decoding, first need to remember the positions of all Alphabetical letters, both in forward and backward order, which is given below.

Positional Value of Alphabets in Forward order

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

Analytical Skills

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

Positional Value of Alphabets in Backward order

A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	12	11	10	9	8	7	6	5	4	3	2	1

The pattern below is very useful to find many types of question in Alphabet Series when we write the last 13 alphabets in front of the first 13 alphabets of the English:

A	B	C	D	E	F	G	H	I	J	K	L	M
Z	Y	X	W	V	U	T	S	R	Q	P	O	N

- We all know there is 26 alphabets in English literature, in which the word **EJOTY** is very helpful in finding the position in letter series.

E	J	O	T	Y
5	10	15	20	25

- For example, if we want to find the position of the alphabet 'S', then as we know that 'T' is 20, so 'S' is $20 - 1 = 19$. Also, we can find the position of an alphabet from the end by subtracting its value from 27. For example, the position of D from the end is $27 - 4 = 23$.

Backward ranking = 27 - Forward ranking

Forward ranking = 27 - Backward ranking

5.1 Letter Coding

In letter coding-decoding the letters of words are replaced by certain other letters according to specific patterns/rules to form a code. In this type of coding-decoding the alphabets of word coded with the help of different operations such as addition, subtraction, interchanging and so on. Candidates need to find the code of another word using the same operations.



Example: If EARTH is coded as FBSUI, how is MOTHER coded as?

Logic: In this, each letter is moved one letter forward. So, the code for the given word MOTHER is NPUIFS.



Example: If COURSE is coded as GSYVWL, how is NATION coded as?

Logic: In this, each letter is moved four letters forward. So, the code for the given word NATION is REXMSR.



Example: In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?

Logic: The letter of the word is written in a reverse order and then each letter is moved one step backward to obtain the code. So, the code for the given word TIGER is **QDFHS**.



Example: In a certain code, PARTNER is written as NCPVLGP. In the same code, what will NATION be written as?

Logic: The letters at odd positions are each moved two steps backward and those at even positions are each moved two steps forward to obtain the corresponding letters of the code.

So, the code for the given word NATION is **LCKRMP**.



Example: If EARTH is written as FCUXM in a certain code. How is TEMPLE written in that code?

Logic: the letters of given word are respectively moved one, two, three, four, and so on steps forward to obtain the corresponding letters of the code.

So, the code for the given word TEMPLE is **UGPTQK**.



Example: If BOMBAY is written as MYMYMY, how will TAMIL NADU be written in that code?

Logic: The letters at the third and sixth places are repeated thrice to code BOMBAY as **MYMYMY**.

Similarly, the letters at the third, sixth and ninth places are repeated thrice to code TAMIL NADU as **MNUMNUMNU**.



Example: If in a certain language, COUNSEL is coded as BITIRAK, how is GUIDANCE written in that code?

Logic: The letters at odd positions are each moved one step backward, while the letters at even positions are respectively moved six, five, four, three, two and so on steps backward to obtain the corresponding letters of the code. So, the code for the given word GUIDANCE is **FOHYZJBFOHYZJBBB**.

Direct Letter Coding

- In direct coding, the code letters occur in the same sequence as the corresponding letters occur in the words.
- Questions on direct-coding, either the particular codes of letters are given or the codes of two or more words are given and one is asked to find the codes of given words involving only those letters for which the codes have already been mentioned.



Example: In a certain code, STOVE is written as FNBLK, then how will VOTES be written in the same code?

Solution:

Letter	S	T	O	V	E
Code	F	N	B	L	K

The code for VOTES is **LBNKF**.



Example: In a code language, STARK is written as LBFMG and MOBILE is written as TNRS PJ. How is BLAME written in that code?

Solution:

Letter	S	T	A	R	K	M	O	B	I	L	E
--------	---	---	---	---	---	---	---	---	---	---	---

Analytical Skills

Code	L	B	F	M	G	T	N	R	S	P	J
------	---	---	---	---	---	---	---	---	---	---	---

Therefore BLAME is written as **RPFTJ**.

5.1.2 Letter Decoding

It is the conversion of the coded numbers or alphabets to the original text. The procedure to decode is the same as coding. That is, find the pattern that is followed in the given series.



Example: If in a certain language NZTUJGZ is coded as MYSTIFY, how is OFNFTJT decoded in that language?

Logic: Each letter in the word NZTUJGZ is moved one step backward to obtain the corresponding letter of the code.

So, in OFNFTJT, O will be coded as N, F as E, N as M and so on. Thus, the code becomes NEMESIS.

5.2 Number/Symbol Coding

- In this, either the numerals are assigned to the alphabets of the given code or the alphabets are assigned to the numerals. The candidate has to observe the direction of solving the problem.
- Clearly, letters and numbers are correlated to each other in no other way except in relation to the position of the letters in the English alphabet.
- In all other cases, the question is one of direct coding.



Example: If in a certain code, TWENTY is written as 863985 and ELEVEN is written as 323039, how are TWELVE written in that code?

Logic: The alphabets are coded as shown:

T	W	E	N	T	Y	L	V
8	6	3	9	8	5	2	0

So, In TWELVE, T is coded as 8, W as 6, E as 3, L as 2 and V as 0. Thus, the code for TWELVE is 863203.



Example: If BRAND is written as 79643 and PAROT is written as 26951, how is PANT coded?

Logic: The alphabets are coded as shown:

B	R	A	N	D	P	O	T
7	9	6	4	3	2	5	1

So, in PANT, P is coded as 2, A as 6, N as 4 and T as 1. Thus, the code for PANT is 2641.



Example: In a certain code, if LOGIC is coded as 1512201824, how is PEARL coded as?

Logic: Each letter's reverse alphabet number. So, PEARL is coded as 112226915.



Example: If APPLE is written as 24991320, how is LOVELY coded as?

Logic: '2' is subtracted from the reverse alphabet numbers of the corresponding letters. Hence, LOVELY is coded as 1310320130.



Example: If $A = 2$, $M = 26$, $Z = 52$, then $BET = ?$

Logic: each letter is assigned a numerical value which is twice the numeral denoting its position in the English alphabet.

B, E and T are 2nd, 5th, and 20th letters respectively.

So, $BET = B + E + T = (2 \times 2) + (5 \times 2) + (20 \times 2) = 54$.



Example: If in a certain code, $BAT = 23$ and $CAT = 24$, then how will you code BALL?

Logic: Taking $A=1$, $B=2$, $C=3$, $D=4$, , $X=24$, $Y=25$, $Z=26$.

We have: $BAT = B + A + T = 2 + 1 + 20 = 23$ and $CAT = C + A + T = 3 + 1 + 20 = 24$

So, $BALL = B + A + L + L = 2 + 1 + 12 + 12 = 27$.



Example: If $GO = 32$, $SHE = 49$, then $SOME$ will be equal to

Logic: In the given code, $Z = 1$, $Y = 2$, $X = 3$, , $C = 24$, $B = 25$, $Z = 26$.

So, $GO = 20 + 12 = 32$ and $SHE = 8 + 19 + 22 = 49$.

Similarly, $SOME = S + O + M + E = 8 + 12 + 14 + 22 = 56$.



Example: If $AT = 20$, $BAT = 40$, then CAT will be equal to

Logic: Taking $A = 1$, $B = 2$, , $T = 20$, , $Z = 26$, we have:

$AT = A \times T = 1 \times 20 = 20$, $BAT = B \times A \times T = 2 \times 1 \times 20 = 40$.

So, $CAT = C \times A \times T = 3 \times 1 \times 20 = 60$.



Example: In a certain code 'CART' is written as '\$ #! &' and PLACE is written as '* @ # \$ %' How can 'CARE' be written in that code?

Logic: From the data we have C as \$, A as #, R as !, T as V, P as *, L as @ and E as %.

So, the code for CARE is \$#! %.



Example: In a certain code, RAID is written as %*#\$, RIPE is written as % * @!. How is DEAR written up in that code?

Logic: The alphabets are coded as shown:

R A I D P E

% # * \$ @ !

So, in DEAR, D as \$, E as !, A as # and R as %. Thus, the code for DEAR is \$! #%

5.3 Substitution

In substitution coding one word is coded with another word. You have to find the right answer for the question and the given code for that word. That will be the right answer for the question.



Example: If 'cook' is called 'butler', 'butler' is called 'manager', 'manager' is called 'teacher', 'teacher' is called 'clerk' and 'clerk' is called 'principal', who will teach in a class?

Logic: Teacher teaches in the class and teacher is coded as Clerk so the clerk will be the write answer for the question.



Example: If 'diamond' is called 'gold', 'gold' is called 'silver', 'silver' is called 'ruby' and 'ruby' is called 'emerald', which is the cheapest jewel?

Logic: The cheapest jewel in the question is silver and silver is coded as the Ruby so ruby will be the right answer.



Example: If white is called blue, blue is called red, red is called yellow, yellow is called green, green is called black, black is called violet and violet is called orange, what would be the colour of human blood?

Logic: The colour of the human blood is 'red' but 'red' is called 'yellow'. So, the colour of human blood is 'yellow'.



Example: If the animals which can walk are called swimmers, animals who crawl are called flying, those living in water are called snakes and those which fly in the sky are called hunters, then what will a lizard be called?

Logic: Clearly, a lizard crawls and the animals that crawl are called 'flying'. So, 'lizard' is called 'flying'.

5.4 Matrix Coding

Matrix is a set of elements laid out in tabular form (in rows and columns). Matrix coding is a method to represent the letters of English alphabet by two digits. One digit is represented by the corresponding row and the other digit is represented by the corresponding column.

In Matrix coding problems of logical reasoning, Normally, a letter from these matrices can be represented first by its row and next by its column. A word is represented by only one set of numbers given in any one of the alternatives.



Example: A word is represented by only one set of numbers as given in any one of the alternatives. The set of numbers given in the alternatives are represented by two classes of alphabets as in two matrices given below. The columns and rows of Matrix I are numbered from 0 to 4 and that Matrix II are numbered from 5 to 9. A letter from these matrices can be represented first by its row and next by its column, e.g., 'A' can be represented by 01, 14, etc. and E can be represented by 55, 66 etc. Similarly, you have to identify the set for the word 'BEST'.

Matrix I

	0	1	2	3	4
0	P	A	G	T	S
1	G	T	S	P	A
2	S	P	A	G	T
3	A	G	T	S	P
4	T	S	P	A	G

Matrix II

	5	6	7	8	9
5	E	M	B	N	O

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6	B	E	O	M	N
7	O	N	E	B	M
8	N	O	M	E	B
9	M	B	N	O	E

(A) 78, 99, 04, 10 (B) 57, 75, 41, 03 (C) 96, 88, 12, 40 (D) 89, 55, 31, 32

Logic: Here, taking the row and column number of each letter, we have

B = 57, 65, 78, 89, 96

E = 55, 66, 77, 88, 99

S = 04, 12, 20, 33, 41

T = 03, 11, 24, 32, 40

Now, taking each option and comparing them with above values, we get only option (C) contains all the correct values.

So, the code for 'BEST' as 96, 88, 12, 40

5.5 Mixed Letter Coding/ Deciphering Message Word Codes

In this type of coding and decoding a few statements consisting of the same words but in different order will be coded as words or letters. Candidates need to find the codes of the words by finding the common words in different statements as the code of a word in both the statements will be the same. In this type of questions, some messages are given in the coded language and the code for a particular word or message is asked.

- To analyse such codes, any two messages bearing a common word are picked up.
- The common code-word will thus represent that word. Proceeding similarly by picking up all possible combinations of two, the entire message can be decoded and the codes for individual words found.

Tips to solve The Question

- Take two sentences at a time and try to find the common words.
- Common words will have common codes in both sentences.



Example: In a certain code language 'pit' 'dar' 'na' means 'you are good'; 'dar' 'tok' 'pa' means 'good and bad'; 'tim' 'na' 'tok' means 'they are bad'. In that language, which word stands for 'they'?

Logic:

- 'Pit' 'dar' 'na' means 'you are good'.
- 'dar' 'tok' 'pa' means 'good and bad'.
- 'tim' 'na' 'tok' means 'they are bad'.

In sentence 1 and 2 'dar' and 'good' are common, therefore 'good' is code for 'dar'.

In sentence 2 and 3 'tok' and 'bad' are common, therefore 'bad' is code for 'tok'.

In sentence 1 and 3 'na' and 'are' is common, therefore 'are' is code for 'na'.

Now in 3rd sentence, we know the code for 'na' and 'tok', therefore code of 'tim' is 'they'.



Example: In a certain code language, 'col' 'tip' 'mot' means 'singing is appreciable', 'mot' 'baj' 'min' means 'dancing is good' and 'tip' 'nop' 'baj' means 'singing and dancing', then,

which of the following means 'good' in that code language?

Logic: In the first and second statements, the common code word is 'mot' and the common word is 'is'. So, 'mot' means 'is'.

In the second and third statements, the common code word is 'baj' and the common word is 'dancing'. So, 'baj' means 'dancing'.

Thus, in the second statements, 'mint' means 'good'.

5.6 Mixed Number Coding/ Deciphering Message Number and Symbol Codes

Mixed number coding is the same as mixed letter coding but instead of alphabetical codes numerical codes are given.

- In this type of questions, a few groups of numbers/symbols, each coding a certain message, are given.
- Through a comparison of the given coded messages, taking two at a time, the candidate is required to find the number/symbol code for each word and then formulate the code for the given message.



Example: In a certain code language, '851' means 'good sweet fruit', '783' means 'good red rose' and '341' means 'rose and fruit'. Which of the following digits stands for 'sweet' in that language?

Logic: In the first and second statements, the common code digit is '8' and the common word is 'good'. So, '8' means 'good'.

In the first and third statements, the common code digit is '1' and the common word is 'fruit'. So, '1' means 'fruit'.

Thus, in the first statement, '5' means 'sweet'.



Example: In a certain code, '467' means 'leaves are green'; '485' means 'green is good' and '639' means 'they are playing'. Which digit stands for 'leaves' in that code?

Logic: In the first and second statements, the common code digit is '4' and the common word is 'green'. So, '4' means 'green'. In the first and third statements, the common code digit is '6' and the common word is 'are'. So, '6' means 'are'.

Thus, in the first statement, '7' means 'leaves'.



Example: In a certain code language, 'pen pencil' is written as '%&', 'eraser sharpener' is written as '@#%' and 'pencil eraser' is written as '%@'. Then, what is the code for 'pen'?

Logic: In the first and third statements, the common code symbol is '%' and the common word is 'pencil'. So, '%' means 'pencil'.

Thus, in the first statement, '&' means 'pen'.

Summary

The key concepts learnt from this Unit are: -

- We have learnt about different types of Coding -Decoding statements.
- We have learnt logics to solve different types of Coding-Decoding problems.

- We have learnt about some set of rules to solve Coding-Decoding problems.

Keywords

- Letter coding
- Number coding
- Substitution coding
- Matrix coding
- Mixed Letter coding
- Mixed number coding

Self Assessment

1. If BROWN is written as 'ZPMUL', then VIOLET is coded as
A. TGMJCR
B. SGMJCQ
C. TGMJCQ
D. TGWCQ
2. If RETURN is coded as PGRWPP, how is SANDLE coded as?
A. UXPBNC
B. QCLFJG
C. CNBPXU
D. CLGFQB
3. In a code language PROPER is coded as NTMRCT, how is RETURN coded in the same code?
A. TCVSTL
B. PGRWPP
C. CNBPXU
D. PGRWQB
4. If E=22, CAT =57 then RAM is equal to
A. 35
B. 30
C. 32
D. 49
5. DEER = 12215 and HIGH = 5654, how will you code HEEL?
A. 2328
B. 3449
C. 4337
D. 5229
6. If ZIP = 198 and ZAP = 246, then how will you code VIP?

Analytical Skills

- A. 174
B. 222
C. 888
D. 990
7. In a certain code 'LAKE' is written as \$ #! & TESLA is written as @&%\$# How can 'TASK' be written in that code?
A. #@\$!
B. @%#\$
C. @#%!
D. None
8. In a code language 'lol sip see' means 'reading is habit', 'see' 'maj' 'rin' means 'playing is good', 'sip' 'cop' 'maj' means 'reading and playing'. Then which of the following means 'good' in that code language?
A. see
B. rin
C. cop
D. maj
9. If in a code language 'rit' 'puc' 'bec' means 'eat fresh food', 'puc' 'tec' 'jac' means 'food is tasty', 'jac' 'lac' 'mac' means 'she is beautiful', then which word means 'tasty'?
A. puc
B. bec
C. rit
D. tec
10. In a certain code, '154' means 'Ram is good'; '537' means 'Shyam is bad' and '472' means 'good and bad'. Which of the following represents 'Shyam' in that code?
A. 5
B. 1
C. 3
D. 2
11. If blue is called red, red is called green, green is called black and black is called white, what is the colour of sky?
A. red
B. black
C. white
D. None of these
12. If SYMBOL is written as NZTMPC in a certain code. How is NUMBER written in that code?
A. NVOSFC

- B. OVFSFC
C. NVOFCS
D. None

13. If AC is called TV, TV is called fridge, fridge is called washing machine, washing machine is called computer, computer is called phone, phone is called printer and printer is called calculator then what would be used to wash clothes?

- A. TV
B. Calculator
C. Computer
D. Printer

14. In a code language if SEVEN is written as 19522514, then in the same code language how EIGHT will be written.

- A. 597820
B. 598718
C. 598207
D. 592087

15. A word is represented by only one set of numbers as given in any one of the alternatives. The sets of numbers given in the alternatives are represented by two classes of alphabets as in two matrices given below. The columns and rows of Matrix I are numbered – from 0 to 4 and that Matrix II are numbered from 5 to 9. A letter from these matrices can be represented first by its row and next by its column, e.g., 'U' can be represented by 01, 14, etc. and E can be represented by 55, 66 etc. Similarly, you have to identify the set for the word 'JUDGE'.

Matrix I

	0	1	2	3	4
0	J	U	G	R	Z
1	G	R	Z	J	U
2	Z	J	U	G	R
3	U	G	R	Z	J
4	R	Z	J	U	G

Matrix II

	5	6	7	8	9
5	E	M	D	N	O
6	D	E	O	M	N
7	O	N	E	D	M
8	N	O	M	E	D

Analytical Skills

9	M	D	N	O	E
---	---	---	---	---	---

- A. 13, 31, 96, 10, 82
 B. 00, 30, 56, 31, 99
 C. 42, 43, 65, 21, 55
 D. 34, 01, 89, 23, 66

Answers for Self Assessment

1. A 2. B 3. B 4. D 5. D
 6. B 7. C 8. B 9. D 10. C
 11. A 12. A 13. C 14. A 15. D

Review Questions

- In a certain language if HOUSE is written as KRUQC then how ROHIT will be written in that language?
- If BRAND is written as 79643 and PAROT is written as 26951, how is PANT coded?
- If SUNDAY is written as NYNYNY, how will BANGALORE be written in that code?
- If EARTH is written as FCUXM in a certain code. How is MOON written in that code?
- In a certain code language, '123' means 'hot filtered coffee', '356' means 'very hot day' and '589' means 'day and night'. Which digit stands for 'very'?
- In a certain code language, 'go for morning walk' is written as '\$*?#', 'good for health' is written as '%?@' and 'good to walk fast' is written as '+@^#', then what is the code for 'health' in that code language?

Further Reading

- A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, S Chand Publishing
- 2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

Web Links

- <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
- 2. <https://www.hitbullseye.com/Reasoning>

Unit 06: Percentage

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Concept of percentage.
- Percentage commodity price increase/decrease.
- Successive percent changes and budget-based problems.

Introduction

The term percent means per hundred or for every hundred. It is the abbreviation of the Latin phrase per centum. Scoring 60 per cent marks means out of every 100 marks the candidate scored 60 marks. The term percent is sometimes abbreviated as p.c. The symbol % is often used for the term percent. Thus, 40 percent will be written as 40%. A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called rate percent, e.g., $\frac{5}{100}$ and 5 percent means the same thing, i.e., 5 parts out of every hundred parts.

- Concept of Percentage: Per means divided and cent means 100. Thus, x percent means x hundredths, written as x%. To express x% as a fraction: We have, $x\% = \frac{x}{100}$.
- To convert any value on the base of hundred.

6.1 Conversion of Fraction to Percentage and Percentage to Fraction

Conversion of fraction to percentage

To convert fraction to percentage multiply the fraction by 100.

For e.g. $\frac{3}{8} = (\frac{3}{8}) \times 100 = 37.5\%$

$\frac{1}{2} = 50\%$					
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{2}{3} = 66\frac{2}{3}\%$				
$\frac{1}{4} = 25\%$	$\frac{3}{4} = 75\%$				
$\frac{1}{5} = 20\%$	$\frac{2}{5} = 40\%$	$\frac{3}{5} = 60\%$	$\frac{4}{5} = 80\%$		
$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{5}{6} = 83\frac{1}{3}\%$				
$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{2}{7} = 28\frac{4}{7}\%$	$\frac{3}{7} = 42\frac{6}{7}\%$	$\frac{4}{7} = 57\frac{1}{7}\%$	$\frac{5}{7} = 71\frac{3}{7}\%$	$\frac{6}{7} = 85\frac{5}{7}\%$
$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$		
$\frac{1}{9} = 11\frac{1}{9}\%$	$\frac{2}{9} = 22\frac{2}{9}\%$	$\frac{4}{9} = 44\frac{4}{9}\%$	$\frac{5}{9} = 55\frac{5}{9}\%$	$\frac{7}{9} = 77\frac{7}{9}\%$	$\frac{8}{9} = 88\frac{8}{9}\%$
$\frac{1}{10} = 10\%$	$\frac{3}{10} = 30\%$	$\frac{7}{10} = 70\%$	$\frac{9}{10} = 90\%$		
$\frac{1}{11} = 9\frac{1}{11}\%$	$\frac{2}{11} = 18\frac{1}{11}\%$	$\frac{3}{11} = 27\frac{3}{11}\%$	$\frac{4}{11} = 36\frac{4}{11}\%$	$\frac{5}{11} = 45\frac{5}{11}\%$	$\frac{6}{11} = 54\frac{6}{11}\%$
			$\frac{7}{11} = 63\frac{7}{11}\%$	$\frac{8}{11} = 72\frac{8}{11}\%$	$\frac{9}{11} = 81\frac{9}{11}\%$
$\frac{1}{12} = 8\frac{1}{3}\%$					

Conversion of percentage to fraction:

To convert percentage to fraction, divide that percent value by 100.

e.g. $50\% = 50/100 = \frac{1}{2}$

$25\% = 25/100 = \frac{1}{4}$



Example 1: What percentage is equivalent to $\frac{3}{5}$?

Solution: $\frac{3}{5} \times 100 = 60\%$. 02 To convert a per cent into a fraction: To convert a percent into a fraction, drop the percent sign and divide the number by 100.



Example 2: What fraction is $16\frac{2}{3}\%$?

Solution: $16\frac{2}{3}\% = \frac{\frac{50}{3}}{100} = \frac{50}{3} \times \frac{1}{100} = \frac{1}{6}$

To find a percentage of a given number: $x\%$ of given number (N) = $\frac{x}{100} \times N$



Example 3: 75% of 400 =?

Solution: 75% of 400 = $\frac{75}{100} \times 400 = 300$.



Example 4: Find a number whose 4% is 72.

Solution: Let the required number be x . Then, 4% of $x = 72$

$$\Rightarrow \frac{4}{100} \times x = 72$$

$$\Rightarrow x = \frac{100}{4} \times 72 = 1800.$$



Example 5: What per cent of 25 Kg is 3.5 Kg?

Solution: Let $x\%$ of 25 Kg be 3.5 Kg.

$$\text{Then, } x\% \text{ of 25 Kg} = 3.5 \text{ Kg} \Rightarrow \frac{x}{100} \times 25 = 3.5$$

$$\Rightarrow x = 3.5 \times 4 = 14.$$

Hence, 3.5 Kg is 14% of 25 Kg.

6.2 Short-cut Methods

01 (a) If A is $x\%$ more than that of B, then B is less than that of A by

$$\left[\frac{x}{100+x} \times 100 \right] \%$$

A

10%

20%

25%

33.33%

50%

B

9.09%

16.66%

20%

25%

33.33%

(b) If A is $x\%$ less than that of B, then B is more than that of A by

$$\left[\frac{x}{100-x} \times 100 \right] \%$$

Explanation

$$\text{Given } A = B + \frac{x}{100} B = \frac{100+x}{100} B$$

$$\therefore A - B = \frac{100+x}{100} B - B = \frac{x}{100} B$$

$$\frac{A-B}{A} = \frac{\frac{x}{100} B}{\frac{100+x}{100} B} = \frac{x}{100+x}$$

$$\text{So, } A-B = \left(\frac{x}{100+x} \times 100 \right) \% \text{ of } A$$

Therefore, B is less than that of A by

$$\left(\frac{x}{100+x} \times 100 \right) \%$$

Similarly, (b) can be proved.



Example 6: If Mohan's salary is 10% more than that of Sohan, then how much per cent is Sohan's salary less than that of Mohan?

Solution: Here, $x = 10$.

$$\therefore \text{Required answer} = \left(\frac{x}{100+x} \times 100 \right) \% = \left(\frac{10}{100+10} \times 100 \right) \% = 9\frac{1}{11} \%$$



Example 7: If A's income is 40% less than B's income, then how much per cent is B's income more than A's income?

Solution: Here, $x = 40$

$$\therefore \text{Required answer} = \left[\frac{x}{100-x} \times 100 \right] \% = \left[\frac{40}{100-40} \times 100 \right] \% = \frac{200}{3} \% = 66\frac{2}{3} \%$$

02 If A is $x\%$ of C and B is $y\%$ of C, then $A = \frac{x}{y} \times 100\%$ of B

Explanation

$$\text{Given } A = \frac{x}{100}C \Rightarrow C = 100 \frac{A}{x}$$

$$\text{And, } B = \frac{y}{100}C \Rightarrow C = 100 \frac{B}{y} \Rightarrow$$

$$\therefore C = 100 \frac{A}{x} = 100 \frac{B}{y}$$

$$\Rightarrow A = \frac{x}{y}B$$

$$\text{Or, } \frac{x}{y} \times 100\% \text{ of } B$$



Example 8: If A is 20% of C and B is 25% of C, then what percentage is A of B?

Solution: Here, $x = 20$ and $y = 25$.

$$A = \frac{x}{y} \times 100\% \text{ of } B$$

$$= \frac{20}{25} \times 100\% \text{ of } B, \text{ i. e., } 80\% \text{ of } B$$

03 (a) If two numbers are respectively $x\%$ and $y\%$ more than a third number, then the first number is

$$\left(\frac{100+x}{100+y} \right) \times 100\% \text{ of the second}$$

and the second number is

$$\left(\frac{100-y}{100-x} \right) \times 100\% \text{ of the first}$$

(b) If two numbers are, respectively, $x\%$ and $y\%$ less than a third number, then the first number is

$$\left(\frac{100-x}{100-y} \right) \times 100\% \text{ of the second}$$

and the second number is

$$\left(\frac{100-y}{100-x} \right) \times 100\% \text{ of the first}$$

Explanation Let A, B and C be the three numbers. Given:

$$A = C + \frac{x}{100}C = \frac{100+x}{100}C$$

$$\Rightarrow C = A \frac{100}{100+x}$$

$$\text{And, } B = C + \frac{y}{100}C = \frac{100+y}{100}C \Rightarrow C = B \frac{100}{100+y}$$

$$\therefore A \frac{100}{100+x} = B \frac{100}{100+y}$$

$$\Rightarrow A = \frac{100+x}{100+y}B \text{ Or } \frac{100+x}{100+y} \times 100\% \text{ of } B$$

$$\text{and, } B = \frac{100+y}{100+x}A \text{ Or } \frac{100+y}{100+x} \times 100\% \text{ of } A$$

Similarly, (b) can be proved



Example 9: Two numbers are respectively 20% and 50% more than a third number. What per cent is the first of the second?

Solution: Here, $x = 20$ and $y = 50$.

$$\therefore \text{First number} = \left(\frac{100+x}{100+y}\right) \times 100\% \text{ of the second} = \left(\frac{100+20}{100+50}\right) \times 100\% \text{ of the second}$$

i.e., 80% of the second.



Example 10: Two numbers are, respectively, 32% and 20% less than a third number. What per cent is the first of the second?

Solution: Here, $x = 32$ and $y = 20$.

$$\therefore \text{First number} = \left(\frac{100-x}{100-y}\right) \times 100\% \text{ of the second} = \left(\frac{100-32}{100-20}\right) \times 100\% \text{ of the second} = \left(\frac{68}{80}\right) \times 100\% \text{ of the second}$$

i.e., 85% of the second.

04 If the price of a commodity increases by $1/n$, then the consumption should be reduced by $1/n+1$ to make the expenditure same.

Expenditure = Price * Consumption

(a) If the price of a commodity increases by $P\%$, then the reduction in consumption so as not to increase the expenditure is

$$\frac{p}{100 + p} \times 100\%$$

(b) If the price of a commodity decreases by $P\%$, then the increase in consumption so as not to decrease the expenditure is

$$\frac{p}{100 - p} \times 100\%$$

Explanation Let the original price of the commodity be Rs.100. Then, the increased price

$$= 100 + \frac{p}{100} \times 100 = \text{Rs. } (100 + P).$$

Therefore, to keep the price unchanged, there should be a reduction in the consumption of the commodity by Rs. P.

Decrease in Rs $(100 + P) = \text{Rs. } P$

$$\therefore \text{Decrease in Rs } 1 = \frac{P}{100+P}$$

$$\therefore \text{Decrease in Rs } 100 = \frac{P}{100+P} \times 100$$

\therefore Required % reduction in consumption is

$$= \frac{P}{100+P} \times 100\%$$

Similarly, (b) part can be proved.



Example 11: If the price of sugar increases by 25%, find how much per cent its consumption be reduced so as not to increase the expenditure.

Solution: Reduction in consumption

$$= \frac{P}{100+P} \times 100\% = \frac{25}{100+25} \times 100\% = 20\%$$



Example 12: If the price of a commodity decreases by 25%, find how much per cent its consumption be increased so as not to decrease the expenditure.

Solution: Increase in consumption

$$= \frac{p}{100-p} \times 100\% = \frac{25}{100-25} \times 100\% = 33\frac{1}{3}\%$$

05 If a number is changed (increased/decreased) successively by $x\%$ and $y\%$, then net % change is given by $\left(x + y + \frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign is +ve or -ve. If x or y indicates decrease in percentage, then put -ve sign before x or y , otherwise +ve sign.

Explanation

Let the given number be N .

If it is increased by $x\%$, then it becomes

$$N + x\% \text{ of } N = N + \frac{Nx}{100} = \frac{N(x+100)}{100}$$

If it is further increased by $y\%$, then it becomes

$$\frac{N(x+100)}{100} + \frac{y}{100} \times \frac{N(x+100)}{100} = \frac{N(x+100)(y+100)}{100^2} = \frac{N(100x+100y+xy+100^2)}{100^2} = \frac{N}{100} \left(x + y + \frac{xy}{100}\right) + N$$

$$\therefore \text{Net change} = \frac{N}{100} \left(x + y + \frac{xy}{100}\right) + N - N = \frac{N}{100} \left(x + y + \frac{xy}{100}\right) = \frac{1}{100} \left(x + y + \frac{xy}{100}\right) \text{ of } N$$

$$\therefore \% \text{ change} = \left(x + y + \frac{xy}{100}\right)\%$$



Example 13: If salary of a person is first increased by 15% and thereafter decreased by 12%, what is the net change in his salary?

Solution: Here, $x = 15$ and $y = -12$.

\therefore The net% change in the salary

$$= \left(x + y + \frac{xy}{100}\right)\% = \left(15 - 12 - \frac{15 \times 12}{100}\right)\% = (3 - 1.8)\% = 1.2\%$$

The +ve, sign indicates the salary of the person increases by 1.2%



Example 14: The population of a town is decreased by 25% and 40% in two successive years. What per cent population is decreased after two years?

Solution: Here, $x = -25$ and $y = -40$.

\therefore The net% change in population

$$= \left(x + y + \frac{xy}{100}\right)\% = \left(-25 - 40 + \frac{25 \times 40}{100}\right)\% = -55\%$$

The -ve sign indicates that there is decrease in population after two years by 55%

06 If two parameters A and B are multiplied to get a product and if A is changed (increased/decreased) by $x\%$ and another parameter B is changed (increased/decreased) by $y\%$, then the net % change in the product ($A \times B$) is also given by

$$\left(x + y + \frac{xy}{100}\right)\%$$

For decrement put -ve sign before x or y , and for increment put +ve sign before x or y .



Example 15: If the side of a square is increased by 20%, its area is increased by $k\%$. Find the value of k .

Solution: Since side \times side = area

\therefore Net% change in area

$$\left(x + y + \frac{xy}{100}\right)\% = \left(20 + 20 + \frac{400}{100}\right)\% = 44\%$$

Therefore, the area is increased by 44%. Here, $k = 44$.



Example 16: The radius of a circle is increased by 2%. Find the percentage increase in its area.

Solution: Since $\pi \times \text{radius} \times \text{radius} = \text{area}$

\therefore Net% change in area

$$= \left(x + y + \frac{xy}{100} \right) \% = \left(2 + 2 + \frac{4}{100} \right) \% = 4\frac{1}{25} \% \text{ [Here, } x = 2 \text{ and } y = 2 \text{]}$$

Therefore, the percentage increase in area is $4\frac{1}{25} \%$



Example 17: The tax on a commodity is diminished by 15% and its consumption increases by 10%.

Find the effect on revenue.

Solution: Since tax \times consumption = revenue

\therefore Net% change in revenue

$$= \left(x + y + \frac{xy}{100} \right) \% = \left(-15 + 10 - \frac{150}{100} \right) \% = -6.5 \% \text{ [Here, } x = -15 \text{ and } y = 10 \text{]}$$

\therefore The revenue decreases by 6.5%

07 If the present population of a town (or value of an item) be P and the population (or value of item) changes at r% per annum, then

(a) Population (or value of item) after n years

$$= p \left(1 + \frac{r}{100} \right)^n$$

(b) Population (or value of item) n years ago

$$= \frac{p}{\left(1 + \frac{r}{100} \right)^n}$$

here r is +ve or -ve according as the population

(Or value of item) increases or decreases.

Explanation

Population at the end of first year

$$= P + \frac{r}{100} P = p \left(1 + \frac{r}{100} \right)$$

Now, the population at the beginning of second year

$$= p \left(1 + \frac{r}{100} \right)$$

\therefore Population at the end of second year

$$= p \left(1 + \frac{r}{100} \right) + p \left(1 + \frac{r}{100} \right) \frac{r}{100} = p \left(1 + \frac{r}{100} \right)^2$$

Similarly, Population at the end of n years = $p \left(1 + \frac{r}{100} \right)^n$



Example 18: The population of a town increases 5% annually. If its present population is 84000, what will it be in 2 years' time?

Solution: Here, P = 84000, r = 5 and n = 2.

\therefore Population of the town after 2 years

$$= p \left(1 + \frac{r}{100} \right)^2 = 84000 \left(1 + \frac{5}{100} \right)^2 = 84000 \times \left(\frac{105}{100} \right)^2 = 92610$$



Example 19: The population of a town increases at the rate of 5% annually. If the present population is 4410, what it was 2 years ago?

Solution: Here, $P = 4410$, $r = 5$ and $n = 2$.

∴ Population of the town 2 years ago

$$= \frac{P}{\left(1 + \frac{r}{100}\right)^n} = \frac{4410}{\left(1 + \frac{5}{100}\right)^2} = \frac{4410}{\left(\frac{21}{20}\right)^2} = \frac{4410}{\frac{441}{400}} = 4000$$

08 If a number A is increased successively by $x\%$ followed by $y\%$ and then by $z\%$, then the final value of A will be

$$= A \left(1 + \frac{x}{100}\right) \left(1 + \frac{y}{100}\right) \left(1 + \frac{z}{100}\right)$$

In case a given value decreases by any percentage, we will use a -ve sign before that.



Example 20: The population of a town is 144000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years?

Solution: Here, $P = 144000$, $x = 5$, $y = -10$ and $z = 15$.

∴ Population of the town after 3 years

$$= A \left(1 + \frac{x}{100}\right) \left(1 + \frac{y}{100}\right) \left(1 + \frac{z}{100}\right) = 144000 \left(1 + \frac{5}{100}\right) \left(1 - \frac{10}{100}\right) \left(1 + \frac{15}{100}\right) = \frac{144000 \times 105 \times 90 \times 115}{100 \times 100 \times 100} = 156492$$

09 In an examination, the minimum pass percentage is $x\%$. If a student secures y marks and fails by z marks, then the maximum marks in the examination is $\frac{100(y+z)}{x}$

Explanation: Let the maximum marks be m . Given: $x\%$ of $m = y + z$

$$\Rightarrow \frac{x}{100} \times m = y + z \text{ or } m = \frac{100(y+z)}{x}$$



Example 21: In an examination, a student must get 60% marks to pass. If a student who gets 120 marks, fails by 60 marks, find the maximum marks.

Solution: Here, $x = 60$, $y = 120$ and $z = 60$.

$$\therefore \text{Maximum marks} = \frac{100(y+z)}{x} = \frac{100(120+60)}{60} = 300$$

10 In an examination $x\%$ and $y\%$ students respectively fail in two different subjects while $z\%$ students fail in both the subjects, then the percentage of students who pass in both the subjects will be

$$[100 - (x + y - z)] \%$$

Explanation

Percentage of students who failed only in first subject = $(x - z) \%$

Percentage of students who failed only in second subject = $(y - z) \%$

Percentage of students who failed in both the subjects = $z\%$

∴ Percentage of students who passed in both the subjects

$$= [100 - \{(x - z) + (y - z) + z\}] \% = [100 - (x + y - z)] \%$$



Example 22: In an examination, 42% students failed in Mathematics and 52% failed in science. If 17% failed in both the subjects, find the percentage of those who passed in both the subjects.

Solution: Here, $x = 42$, $y = 52$ and $z = 17$.

∴ Percentage of students passing both the subjects = $[100 - (x + y - z)] \%$

$$= [100 - (42 + 52 - 17)] \% = 23\%.$$

6.3 Successive Percentage Change

What is Successive Percentage Change?

The concept of successive percentage change deals with two or more percentage changes applied to quantity consecutively. In this case, the final change is not the simple addition of the two percentage changes (as the base changes after the first change).

Formula for Percentage Change:

Suppose a number N undergoes a percentage change of x% and then y%, the net change is:

$$\text{New number} = N \left(1 + \frac{x}{100}\right) \left(1 + \frac{y}{100}\right)$$

$$\text{Now, } \left(1 + \frac{x}{100}\right) \left(1 + \frac{y}{100}\right) = 1 + \frac{x}{100} + \frac{y}{100} + \frac{xy}{10000}$$

$$\text{If we say that } x + y + \frac{xy}{100} = z, \text{ then } \left(1 + \frac{x}{100}\right) \left(1 + \frac{y}{100}\right) = 1 + \frac{z}{100}$$

Here, z is the effective percentage change when a number is changed successively by two percentage changes.

Various cases for Percentage Change:

Both percentage changes are positive:

$$x \text{ and } y \text{ are positive and net increase} = \left(x + y + \frac{xy}{100}\right)\%$$

One percentage change is positive and the other is negative:

$$x \text{ is positive and } y \text{ is negative, then net percentage change} = \left(x - y - \frac{xy}{100}\right)\%$$

Both percentage changes are negative:

$$x \text{ and } y \text{ both are negative and imply a clear decrease} = \left(-x - y + \frac{xy}{100}\right)\%$$

Percentage Change involving three changes:

If value of an object/number P is successively changed by x%, y% and then z%, then final value.

$$= P \left(1 \pm \frac{x}{100}\right) \left(1 \pm \frac{y}{100}\right) \left(1 \pm \frac{z}{100}\right)\%$$



Example 23: A's salary is increased by 10% and then decreased by 10%. The change in salary is

Solution:

$$\text{Percentage change formula when } x \text{ is positive and } y \text{ is negative} = \left(x - y - \frac{xy}{100}\right)\%$$

Here, $x = 10$, $y = 10$

$$= \left(10 - 10 - \frac{100}{100}\right)\% = -1\%$$

As negative sign shows a decrement, hence the final salary is decreased by 1%.

6.4 Budget based Problems



Example 24: Raju spends 40% of his salary on house rent. On the remaining 10% spend on travel. On remaining $16\frac{2}{3}\%$ spend on food and remaining is saved. If he saved Rs.2250, then find the money he spent on food.

Solution:

Let the Raju total salary is Rs. 100. Amount spend on house Rent = 40% of 100 = Rs. 40.

Therefore, remaining amount = 100 - 40 = Rs. 60. Amount spend on travel = 10% of 60 = Rs. 6.

Therefore, remaining amount after spending on travel = 60 - 6 = Rs. 54.

Analytical Skills

Amount spend on food = $16\frac{2}{3}\%$ of 54 = $\frac{50}{300} \times 54 = \text{Rs. } 9$.

Therefore, remaining amount after spending on food = $54 - 9 = \text{Rs. } 45$.

Total saving is 45 out of assumed total salary 100

Therefore, actual total salary = $\frac{100}{45} \times \text{Actual saving} = \frac{100}{45} \times 2250 = \text{Rs. } 5000$.

And money spent on food = $\frac{9}{100} \times 5000 = \text{Rs. } 450$

Summary

The key concepts learned from this unit are: -

- We have learnt percentage commodity price increase/decrease
- We have learnt successive percent changes and budget-based problems.

Keywords

- Percentage
- Commodity
- Successive percent
- Budget

Self Assessment

1. Express 56% as a fraction:
A. (a) $14/25$
B. (b) $18/25$
C. (c) $19/25$
D. (d) $20/25$
2. Express 4% as a fraction:
A. $1/24$.
B. $1/20$.
C. $3/25$.
D. $1/25$.
3. Express 0.6% as a fraction:
A. $3/70$
B. $3/125$
C. $13/50$
D. $3/500$
4. Express 0.008% as a fraction:
A. $12/1250$
B. $13/1250$
C. $1/1250$

D. $7/1250$

5. Express 6% as a Decimal:

- A. 0.06.
- B. 0.28.
- C. 0.002.
- D. 0.004

6. Express 28% as a Decimal:

- A. 0.06.
- B. 0.28.
- C. 0.002.
- D. 0.004

7. Express 0.04% as a Decimal :

- A. 0.06.
- B. 0.28.
- C. 0.002.
- D. 0.004

8. 2 is what percent of 50?

- A. 4%
- B. 12%
- C. 0.2%
- D. 0.04%

9. ? % of 25 = 20125.

- A. 12.5
- B. 9
- C. 8.5
- D. 14.5

10. Which is greatest in $16\frac{2}{3}\%$, $2/5$ and 0.17?

- A. $2/5$
- B. 16
- C. $2/3$
- D. 0.17

11. An inspector rejects 0.08% of the meters as defective. How many will be examined to project?

- A. 2400
- B. 2500
- C. 3000
- D. 4000

12. Sixty five percent of a number is 21 less than four fifth of that number. What is the number?

- A. 130
- B. 140
- C. 150
- D. 200

13. In expressing a length 810472 km as nearly as possible with three significant digits, find the percentage error.

- A. 0.0034
- B. 0.034
- C. 0.34
- D. 3.4

14. if 50% of $(x-y) = 30\%$ of $(x+y)$ then what percent of x is y ?

- A. 25%
- B. 50%
- C. 75%
- D. 100%

15. The salary of a person was reduced by 10%. By what percent should his reduced salary be raised so as to bring it at par with his original salary?

- A. $(100/9)\%$
- B. $(20/9)\%$
- C. $(100/19)\%$
- D. $(100/29)\%$

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. A | 2. D | 3. D | 4. C | 5. A |
| 6. B | 7. D | 8. A | 9. C | 10. D |
| 11. B | 12. B | 13. B | 14. A | 15. A |

Review Questions

1. When the price for a product was decreased by 10%, the number sold increased by 30%. What was the effect on the total revenue?
2. If the numerator of a fraction be increased by 15% and its denominator be diminished by 8%, the value of the fraction is $15/16$. Find the original fraction.
3. The population of a town is 176400. If it increases at the rate of 5% per annum, what will be its population 2 years hence? What was it 2 years ago?
4. In the new budget, the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?
5. The value of a machine depreciates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years? What was the value of the machine 2 years ago?

6. During one year, the population of town increased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year?
7. If A earns 99/3% more than B, how much percent does B earn less than A?
8. How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution?
9. In an examination, 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.
10. The salary of a person was reduced by 20%. By what percent should his reduced salary be raised so as to bring it at par with his original salary?
11. The capacity of a ground was 100000 at the end of 2012. In 2013, it increased by 10% and in 2014, it decreased by 18.18%. What was the ground's capacity at the end of 2014?



Further Reading

- Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
- A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
- Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
- 4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

Unit 07: Profit and Loss

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Concept of Cost Price, Selling Price and Profit or Gain.
- Understand relation between cost price selling Price and discount.
- Understand Successive discount.
- Solve problems based on cost price, selling price, profit/loss percentage and discount-based problems.

Introduction

Nowadays, transactions have become a common feature of life. When a person deals in the purchase and sale of any item, he either gains or loses some amount generally. The aim of any business is to earn profit.

Profit (P)

The amount gained by selling a product with more than its cost price.

Loss (L)

The amount that you lose after selling a product with less than its cost price.

Cost Price – This is the rate at which the commodity is bought. The amount paid for a product or commodity to purchase it is called a cost price. It can be divided into two parts: (Fixed Cost & Variable Cost)

Selling Price – This is the rate at which the commodity is sold.

Profit – When the Cost Price of the commodity is less than the Selling Price.

Profit = Selling Price – Cost Price

Loss – When the Cost Price of the commodity is greater than the Selling Price.

Loss = Cost Price – Selling Price

Analytical Skills

If there is a PROFIT of $x\%$, the calculating figures would be 100 and $(100 + x)$. If there is a LOSS of $x\%$, the calculating figures would be 100 and $(100 - x)$.

Calculating figures be Cost Price and Selling Price respectively.

The commonly used terms in dealing with questions involving sale and purchase are:

Cost Price: The cost price of an article is the price at which an article has been purchased. It is abbreviated as C.P.

Selling Price: The selling price of an article is the price at which an article has been sold. It is abbreviated as S.P.

Profit or Gain: If the selling price of an article is more than the cost price, there is a gain or profit. Thus, Profit or Gain = S.P. - C.P.

$$\text{Profit or Gain \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\% = \frac{\text{Profit}}{\text{C.P.}} \times 100\%$$

Loss: If the cost price of an article is greater than the selling price, the seller suffers a loss. Thus,

Loss = C.P. - S.P. respect to the cost price of the item.

$$\text{Loss \%} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100\% = \frac{\text{Loss}}{\text{C.P.}} \times 100\%$$

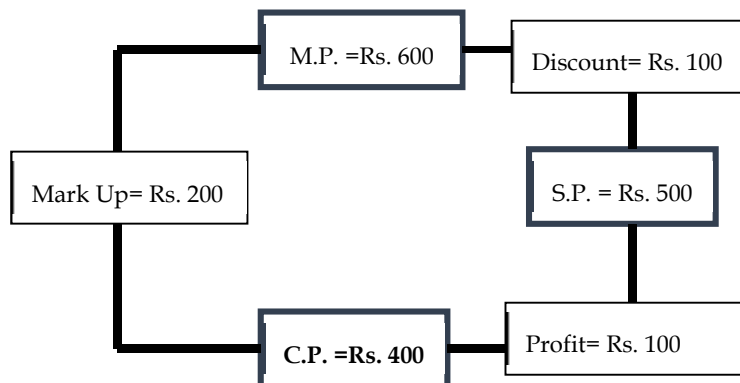
Marked Price Formula (MPF)

Discount = Marked price - selling price

Discount percentage = $(\text{Discount} / \text{Marked price}) \times 100$

For understanding relation between different parameters of profit and loss. Let us take an example

A shopkeeper buys a T-shirt from whole sale market at Rs.400 and marked that article Rs.200 above the CP. After giving a discount of Rs.100 he sold it at Rs.500.



Example 1: (i) If C.P. = 235, S.P. = `240, then profit=?

(ii) If C.P. = 116, S.P. = 107, then loss =?

Solution: (i) Profit = S.P. - C.P. = 240 - 235 = Rs.5.

(ii) Loss = C.P. - S.P. = 116 - 107 = Rs.9.



Another Example: A toy that cost 80 rupees is sold at a profit of 20 rupees. Find the percent or rate of profit.

Solution: Gain/cost $\times 100$ = % profit.

$20/80 \times 100 = 25\%$. - Answer



Example 2: The cost price of a shirt is Rs.200 and selling price is Rs. 250. Calculate the % of profit.

Solution: We have, C.P. = Rs.200, S.P. = Rs.250.

Profit = S.P. - C.P. = 250 - 200 = Rs.50.

$$\therefore \text{Profit}\% = \frac{S.P. - C.P.}{C.P.} \times 100\% = \frac{\text{Profit}}{C.P.} \times 100\% = \frac{50}{200} \times 100\% = 25\%$$



Example 3: Anu bought a necklace for rs.750 and sold it for Rs.675. Find her loss %.

Solution: Here, C.P. = Rs.750, S.P. = Rs.675

Loss = C.P. - S.P. = 750 - 675 = Rs.75.

$$\therefore \text{loss}\% = \frac{C.P. - S.P.}{C.P.} \times 100\% = \frac{75}{750} \times 100\% = 10\%$$

When the selling price and gain% are given:

$$C. P. = \left(\frac{100}{100 + \text{Gain}\%} \right) S. P.$$

When the cost price and gain per cent are given:

$$S. P. = \left(\frac{100 + \text{Gain}\%}{100} \right) C. P.$$

Explanation

$$\text{Since profit}\% = \frac{\text{Profit}}{C.P.} \times 100$$

$$\frac{\text{Profit}}{100} \% = \frac{S.P. - C.P.}{C.P.} = \frac{S.P.}{C.P.} - 1 \text{ Or } \frac{S.P.}{C.P.} = 1 + \frac{\text{Profit}\%}{100}$$

$$\therefore S.P. = \left(\frac{100 + \text{Profit/Gain}\%}{100} \right) C. P.$$

$$\text{and, } C.P. = \left(\frac{100}{100 + \text{Gain}\%} \right) S. P.$$

When the cost and loss per cent are given:

$$s. p = \left(\frac{100 - \text{Loss}\%}{100} \right) C. P.$$

When the selling price and loss per cent are given:

$$C.P. = \left(\frac{100}{100 - \text{Loss}\%} \right) S. P.$$

Explanation

$$\text{Since loss}\% = \frac{\text{Loss}}{C.P.} \times 10 = \frac{C.P. - S.P.}{C.P.} \times 100$$

$$\therefore \frac{\text{loss}\%}{100} = 1 - \frac{S.P.}{C.P.}$$

$$\text{Or } \frac{S.P.}{C.P.} = 1 - \frac{\text{loss}\%}{100}$$

$$\therefore S.P. = \left(\frac{100 - \text{loss}\%}{100} \right) C. P.$$

$$\text{And, } C. P. = \left(\frac{100}{100 - \text{loss}\%} \right) S. P.$$



Another Example: A damaged chair that cost Rs.110 was sold at a loss of 10%. Find the loss and the selling price.

Solution: Cost x percent loss = loss. $110 \times 1/10 = 11$, loss. Cost - loss = selling price. $110 - 11 = 99$, selling price.



Example 4: Mr Sharma buys a cooler for Rs.4500. For how much should he sell it to gain 8%?

Solution: We have, C.P. = Rs.4500, gain%=8%

$$\therefore S.P. = \left(\frac{100 + \text{Gain}\%}{100} \right) C. P. = \left(\frac{100 + 8\%}{100} \right) 4500 = \frac{108}{100} \times 4500 = \text{Rs. } 4860$$



Example 5: By selling a fridge for Rs.7200, Pankaj loses 10%. Find the cost price of the fridge.

Solution: We have, S.P. = Rs.7200, loss% = 10%

$$\therefore \text{C.P.} = \left(\frac{100}{100 - \text{loss}\%} \right) \text{S.P.} = \left(\frac{100}{100 - 10} \right) 7200 = \frac{100}{90} \times 7200 = \text{Rs. } 8000$$

To find the profit and the cost when the selling price and the percent profit are given, multiply the selling price by the percent profit and subtract the result from the selling price.



Another Example: A toy is sold for Rs. 6.00 at a profit of 25% of the selling price. Separate this selling price into cost and profit.

Solution:

Selling price \times profit% = profit.

$$\text{Or, profit} = 6.00 \times .25 = 1.50$$

Selling price = profit + cost.

$$\text{Or, Cost price} = 6.00 - 1.50 = 4.50$$



Example 6: By selling a pen for Rs.99, Mohan gains $12\frac{1}{2}\%$. Find out cost price of the pen.

Solution: Here, S.P. = Rs.99, gain% = $12\frac{1}{2}\% = \frac{25}{2}\%$.

$$\therefore \text{C.P.} = \left(\frac{100}{100 + \text{Gain}\%} \right) \text{S.P.} = \left(\frac{100}{100 + \frac{25}{2}} \right) \times 99 = \frac{100 \times 2}{225} \times 99 = \text{Rs. } 88$$

7.1 Some Useful Short Cut Methods

01 If a man buys x items for y and sells z items for w , then the gain or loss per cent made by him is

$$\left(\frac{xw}{zy} - 1 \right) \times 100$$

Explanation

S.P. of z items = Rs. w

S.P. of x items = Rs. $\frac{xw}{z}$

Net profit = Rs. $\frac{xw}{z} - y$

$$\therefore \% \text{profit} = \frac{\text{Rs. } \frac{xw}{z} - y}{y} \times 100 = \left(\frac{xw}{zy} - 1 \right) \times 100\%$$

If the result is negative. Then it will represent loss.



Example 7: If 11 oranges are bought for 10 and sold at 10 for Rs.11, what is the gain or loss %?

Solution:

$$\therefore \% \text{profit} = \left(\frac{xw}{zy} - 1 \right) \times 100\% = \left(\frac{11 \times 11}{10 \times 10} - 1 \right) \times 100\% = 21\%$$

Quantity	Price
11	10
10	11



Example 8: A fruit seller buys apples at the rate of Rs.12 per dozen and sells them at the rate of 15 for Rs.12. Find out his percentage gain or loss.

Solution:

$$\% \text{gain or loss} = \left(\frac{xw}{zy} - 1 \right) \times 100\% = \left(\frac{12 \times 12}{15 \times 12} - 1 \right) \times 100\% = -25\%$$

The -ve sign indicates a loss of 25%.

02 If the cost price of m articles is equal to the selling price of n articles, then (Also applicable for false weight questions)

$$\% \text{gain or loss} = \frac{m-n}{n} \times 100$$

[If $m > n$, it is % gain and, if $m < n$, it is % loss]

Explanation

Let, the C.P. of an article be Rs.1.

$$\therefore \text{C.P. of } m \text{ articles} = m \times 1 = \text{Rs. } m$$

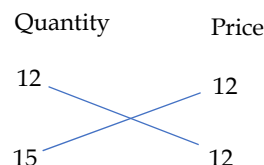
$$\text{S.P. of } n \text{ articles} = \text{Rs. } m$$

$$\therefore \text{S.P. of an article} = \text{Rs. } \frac{m}{n}$$

$$\therefore \text{Profit on 1 article} = \text{Rs. } \left(\frac{m}{n} - 1 \right)$$

$$\text{i.e., Rs. } \left(\frac{m-n}{n} \right)$$

$$\therefore \% \text{profit} = \left(\frac{m-n}{n} \right) \times 100$$



Example 9: A shopkeeper professes to sell his goods on cost price, but uses 800 gm, instead of 1 Kg. What is his gain %?

Solution: Here, cost price of 1000 gm is equal to selling price of 800 gm,

$$\therefore \% \text{gain} = \left(\frac{m-n}{n} \right) \times 100 = \left(\frac{1000-800}{800} \right) \times 100 = \left(\frac{200}{800} \right) \times 100 = 25\%$$



Example 10: If the selling price of 12 articles is equal to the cost price of 18 articles, what is the Profit%?

Solution: Here, $m = 18$, $n = 12$

$$\therefore \text{Profit \%} = \left(\frac{m-n}{n} \right) \times 100 = \left(\frac{18-12}{12} \right) \times 100 = \left(\frac{6}{12} \right) \times 100 = 50\%$$

03 If an article is sold at a price S.P.1, then % gain or % loss is x and if it is sold at a price S.P.2, then % gain or %loss is y . If the cost price of the article is C.P., then

$$\frac{\text{S.P.1}}{100+x} = \frac{\text{S.P.2}}{100+y} = \frac{\text{C.P.}}{100} = \frac{\text{S.P.1} - \text{S.P.2}}{x-y}$$

If x or y is -ve, then it will indicate a loss, and if +ve then a gain or profit.



Example 11: By selling a radio for Rs. 1536, Suresh lost 20%. What per cent shall he gain or lose by selling it for Rs. 2000?

Solution: Here, $\text{S.P.1} = 1536$, $x = -20$ (-ve sign indicates loss)

$$\text{S.P.2} = \text{Rs.2000}, y = ?$$

Using the formula:

$$\frac{\text{S.P.1}}{100+x} = \frac{\text{S.P.2}}{100+y}$$

we get,

$$\frac{1536}{100-20} = \frac{2000}{100+y}$$

$$\Rightarrow 100+y = \frac{2000 \times 80}{1536} = 104\frac{1}{6} \Rightarrow y = 4\frac{1}{6}\%$$

Thus, Suresh has a gain of $4\frac{1}{6}\%$ by selling it for Rs.2000

Analytical Skills

Alternatively,

$$S.P.1 = 1536 = 0.8 \times C.P.1 \text{ (Since 20\% Loss)}$$

$$\Rightarrow C.P.1 = \frac{1536}{0.8} = 1920$$

$$\text{Profit\%} = \frac{S.P. - C.P.}{C.P.} \times 100\% = \frac{2000 - 1920}{1920} \times 100\% = 4\frac{1}{6}\%$$

04 If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%. If 'C' pays `z for it to 'B', then the cost price for 'A' is

$$\left[\frac{100^2 Z}{(100+m)(100+n)} \right]$$

If m or n is -ve, then it will indicate a loss, and if +ve then a gain or profit.



Example 12: Mohit sells a bicycle to Rohit at a gain of 10% and Rohit again sells it to Jyoti at a profit of 5%. If Jyoti pays Rs.462 to Rohit, what is the cost price of the bicycle for Mohit?

Solution: Here, m = 10, n = 5, z = Rs.462.

Using the formula,

$$C.P. = \left[\frac{100^2 Z}{(100+m)(100+n)} \right] = \left[\frac{100^2 \times 462}{(100+10)(100+5)} \right] = \left[\frac{100^2 \times 462}{(110)(105)} \right] = \text{Rs. } 400$$

Alternatively

$$S.P. \text{ of Mohit} = 1.1 \text{ C.P. of Mohit} = C.P. \text{ of Rohit}$$

$$S.P. \text{ of Rohit} = 1.05 \text{ C.P. of Rohit} = 1.05 \times 1.1 \text{ C.P. of Mohit} = 462$$

$$C.P. \text{ of Mohit} = \frac{462}{1.05 \times 1.1} = \text{Rs. } 400$$



Example 13: 'A' sells a DVD to 'B' at a gain of 17% and 'B' again sells it to 'C' at a loss of 25%. If 'C' pays `1053 to 'B', what is the cost price of the DVD to 'A'?

Solution: We have, m = 17, n = -25, z = Rs.1053.

\therefore Cost price of DVD to

$$= \left[\frac{100^2 Z}{(100+m)(100+n)} \right] = \left[\frac{100^2 \times 1053}{(100+17)(100-25)} \right] = \left[\frac{100^2 \times 1053}{(117)(75)} \right] = \text{Rs. } 1200$$

Alternatively

$$S.P._A = 1.17 \times C.P._A = C.P._B$$

$$S.P._B = 0.75 \times C.P._B = 0.75 \times 1.17 \times C.P._A = \text{Rs. } 1053$$

$$\Rightarrow C.P._A = \frac{1053}{0.75 \times 1.17} = \text{Rs. } 1200$$

05 If 'A' sells an article to 'B' at a gain/loss of m%, and 'B' sells it to 'C' at a gain/loss of n%, then the resultant Profit/loss percent is given by

$$\left(m + n + \frac{mn}{100} \right) \%$$

For loss m or n is -ve and for profit it is taken as +ve.



Example 14: 'A' sells a horse to 'B' at a profit of 5% and 'B' sells it to 'C' at a profit of 10%. Find out the resultant profit percent.

Solution: We have, m = 5 and n = 10.

$$\therefore \text{Resultant profit \%} = \left(m + n + \frac{mn}{100} \right) \% = \left(5 + 10 + \frac{50}{100} \right) \% = 15\frac{1}{2} \%$$



Example 15: Manoj sells a shirt to Yogesh at a profit of 15%, and Yogesh sells it to Suresh at a loss of 10%. Find the resultant profit or loss.

Solution: Here, $m = 15$, $n = -10$

$$\therefore \text{Resultant profit/loss \%} = \left(m + n + \frac{mn}{100}\right)\% = \left(15 - 10 - \frac{150}{100}\right)\% = 3\frac{1}{2}\%$$

The +ve sign indicates profit.

06 When two different articles are sold at the same selling price, getting gain/loss of $x\%$ on the first and gain/loss of $y\%$ on the second, then the overall % gain or % loss in the transaction is given by

$$\left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

The above expression represents overall gain or loss according as its sign is +ve or -ve.

07 When two different articles are sold at the same selling price getting a gain of $x\%$ on the first and loss of $x\%$ on the second, then the overall % loss in the transaction is given by

$$\left(\frac{x}{10}\right)^2\%$$

Note that in such questions, there is always a loss.



Example 16: Mahesh sold two scooters, each for ₹24000. If he makes 20% profit on the first and 15% loss on the second, what is his gain or loss per cent in the transactions?

Solution: Here, $x = 20$ and $y = -15$.

$$\therefore \text{Overall gain/loss \%} = \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\% = \left[\frac{100(20-15)-600}{(100+20)+(100-15)}\right]\% = \frac{-100}{205}\% = \frac{-20}{45}\%$$



Example 17: Rajesh sold two horses for ₹990 each; gaining 10% on the one and losing 10% on the other. Find out his total gain or loss per cent.

Solution: Here, $x = 10$.

$$\therefore \text{Overall loss \%} = \left(\frac{x}{10}\right)^2\% = \left(\frac{10}{10}\right)^2\% = 1\% \text{ Loss}$$

Alternatively,

$$S.P._1 = S.P._2 = 1.1C.P._1 = 0.9C.P._2$$

$$\text{Profit/Loss} = S.P._1 + S.P._2 - C.P._1 - C.P._2 = 1.1C.P._1 + 0.9C.P._2 - C.P._1 - C.P._2 = 0.1C.P._1 - 0.1C.P._2$$

$$\text{Profit/Loss \%} = \frac{0.1C.P._1 - 0.1C.P._2}{C.P._1 + C.P._2} \times 100 = \frac{0.1C.P._1 - 0.1 \times \frac{1.1}{0.9}C.P._1}{C.P._1 + \frac{1.1}{0.9}C.P._1} \times 100 = -1\%$$

08 A merchant uses faulty measure and sells his goods at gain/loss of $x\%$. The overall % gain/loss(g) is given by

$$\frac{100 + g}{100 + x} = \frac{\text{True measure}}{\text{Faulty measure}}$$

09 A merchant uses $y\%$ less weight/length and sells his goods at gain/loss of $x\%$. The overall % gain/ loss is given by

$$\left[\left(\frac{y+x}{100-y}\right) \times 100\right]\%$$

7.2 False Weight



Example 18: A dishonest shopkeeper professes to sell cloth at the cost price, but he uses faulty meter rod. His meter rod measures 95 cm only. Find his gain per cent.

Solution: Here, true measure = 100 cm.

False measure = 95 cm.

Analytical Skills

Since the shopkeeper sells the cloth at cost price, $\therefore x = 0$.

\therefore Overall gain % is given by

$$\begin{aligned}\frac{100 + g}{100 + x} &= \frac{\text{True measure}}{\text{Faulty measure}} \\ \Rightarrow \frac{100 + g}{100 + 0} &= \frac{100}{95} \\ \Rightarrow 100 + g &= \frac{100 \times 100}{95} \\ \Rightarrow g &= \frac{10000}{95} - 100 = \frac{500}{95} = \frac{100}{19} = 5\frac{5}{19}\%\end{aligned}$$



Example 19: A dishonest shopkeeper professes to sell his goods at cost price, but he uses a weight of 800 g for the Kg weight. Find out his gain per cent.

Solution: True measure = 1000 g. False measure = 800 g. Also, $x = 0$.

\therefore Overall gain % is given by

$$\begin{aligned}\frac{100 + g}{100 + x} &= \frac{\text{True measure}}{\text{Faulty measure}} \\ \Rightarrow \frac{100 + g}{100 + 0} &= \frac{1000}{800} \\ \Rightarrow 100 + g &= \frac{1000 \times 100}{800} = 125 \\ \Rightarrow g &= 125 - 100 = 25\%\end{aligned}$$



Example 20: A shopkeeper sells goods at 44% loss on cost price, but uses 30% less weight. What is his percentage profit or loss?

Solution: Here, $x = -44$ and $y = 30$.

$$\therefore \text{Overall gain/loss \%} = \left[\left(\frac{y+x}{100-y} \right) \times 100 \right] \% = \left[\left(\frac{30-44}{100-30} \right) \times 100 \right] \% = \frac{-14}{70} \times 100\% = -20\%$$

The negative sign indicates loss of 20%.

10 A person buys two items for A and sells one at a loss of 1% and the other at a gain of g%. If each item was sold at the same price, then

(a) The cost price of the item sold at loss

$$= \frac{A(100 + \text{gain}\%)}{(100 - \text{loss}\%) + (100 + \text{gain}\%)}$$

(b) The cost price of the item sold at gain

$$= \frac{A(100 - \text{loss}\%)}{(100 - \text{loss}\%) + (100 + \text{gain}\%)}$$



Example 21: Ramesh buys two books for Rs.410. He sells one at a loss of 20% and the other at a gain of 25%. If both the books are sold at the same price, find out the cost price of two books.

Solution: Cost price of the book sold at a loss of 20%

$$= \frac{A(100 + \text{gain}\%)}{(100 - \text{loss}\%) + (100 + \text{gain}\%)} = \frac{410(100 + 25)}{(100 - 20) + (100 + 25)} = \frac{410 \times 125}{80 + 125} = \text{Rs. } 250$$

Cost price of the book sold at a profit of 25%

$$= \frac{A(100 - \text{loss}\%)}{(100 - \text{loss}\%) + (100 + \text{gain}\%)} = \frac{410(100 - 20)}{(100 - 20) + (100 + 25)} = \frac{410 \times 80}{80 + 125} = \text{Rs. } 160$$

Alternatively,

Let the cost price of the item sold at loss of 20% = x

Therefore, the cost price of the item sold at profit = 410 - x

From the question

$$S.P._1 = S.P._2 = 0.8x = 1.25(410 - x) = 512.5 - 1.25x$$

$$\Rightarrow 2.05x = 512.5$$

$$\Rightarrow x = C.P._1 = \frac{512.5}{2.05} = \text{Rs. } 250 \text{ and } C.P._2 = 410 - 250 = \text{Rs. } 160$$

7.3 Successive Discount and Marked Price

11 If two successive discounts on an article are $m\%$ and $n\%$ respectively, then a single discount equivalent to the two successive discounts will be:

$$\left(m + n - \frac{mn}{100}\right)\%$$

Explanation

Let, the marked price of the article be Rs.100.

$$\therefore \text{S.P. after the first discount} = \text{Rs. } (100 - m) \text{ and discount at } n\% \text{ on Rs. } (100 - m) = \frac{\text{Rs.}(100 - m) \times n}{100}$$

\therefore Single equivalent discount

$$= \left[m + \frac{(100 - m) \times n}{100}\right]\% = \left(\frac{100m + 100n - mn}{100}\right)\% = \left(m + n - \frac{mn}{100}\right)\%$$



Example 22: Find a single discount equivalent to two successive discounts of 10% and 20%.

Solution: The equivalent single discount is given by

$$\left(m + n - \frac{mn}{100}\right)\% = \left(10 + 20 - \frac{200}{100}\right)\% = 28\%$$

Alternatively,

$$S.P = M.P. \times D_1 \times D_2 = M.P. \times 0.9 \times 0.8 = M.P. \times 0.72$$

Therefore, the equivalent single discount is $(1 - 0.72) \times 100 = 28\%$



Example 23: Find out a single discount equivalent to three successive discounts of 10%, 20% and 30%.

Solution: The equivalent single discount is given by

$$S.P = M.P. \times D_1 \times D_2 \times D_3 = M.P. \times 0.9 \times 0.8 \times 0.7 = M.P. \times 0.504$$

Therefore, the equivalent single discount is $(1 - 0.504) \times 100 = 49.6\%$



Example 24: Two shopkeepers sell machines at the same list price. The first allows two successive discounts of 30% and 16% and the second 20% and 26%. Which discount series is more advantageous to the buyers?

Solution: A single discount equivalent to the two successive discounts of 30% and 16% is

$$S.P = M.P. \times D_1 \times D_2 = M.P. \times 0.7 \times 0.84 = M.P. \times 0.588$$

Therefore, the equivalent single discount is $(1 - 0.588) \times 100 = 41.2\%$

Also, a single discount equivalent to the two successive discounts of 20% and 26% is

$$S.P = M.P. \times D_1 \times D_2 = M.P. \times 0.8 \times 0.74 = M.P. \times 0.592$$

Therefore, the equivalent single discount is $(1 - 0.592) \times 100 = 40.8\%$

Clearly, the discount series being offered by the first shopkeeper is more advantageous to the buyers.

12 A shopkeeper sells an item at Rs. Z after offering a discount of $D\%$ on labelled price. Had he not offered the discount; he would have earned a profit of $P\%$ on the cost price.

Analytical Skills

The cost price of each item is given by

$$C. P = \left[\frac{100^2 Z}{(100-D)(100+P)} \right]$$



Example 25: A shopkeeper sold sarees at Rs. 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a Profit of 12% on the cost price. What was the cost price of each saree?

Solution: We have, labelled price z = Rs.266, discount D = 5% and profit P = 12%

Using the formula

$$C. P = \left[\frac{100^2 Z}{(100-D)(100+P)} \right] = \left[\frac{100^2 \times 266}{(100-5)(100+12)} \right] = \text{Rs. } 250$$

Alternatively,

S.P. = Rs. 266 = $0.95 \times M. P.$ (With 5% discount)

$$M. P. = \frac{266}{0.95} = \text{Rs. } 280 = S. P. (Without discount) = 1.12 \times C. P.$$

Therefore, cost price is $= \frac{280}{1.12} = \text{Rs. } 250$

Summary

The key concepts learned from this unit are: -

- We have learnt how to calculate Cost Price, Selling Price and Profit or Gain.
- We have learnt how to calculate successive discount and marked price

Keywords

- Cost Price
- Selling Price
- Profit
- Gain.
- Discount price
- Marked price

Self Assessment

1. A man buys an article for Rs.27.50 and sells it for Rs.28.50. His gain % is?
A. 1.
B. 2.
C. 3.
D. 4.
2. If a radio is sold for Rs. 490 and sold for Rs. 465.50. The loss% is?
A. 2
B. 3
C. 4
D. 5
3. S.P when CP=56.25, gain=20% is?

- A. 67
B. 68
C. 67.5
D. 68
4. CP when SP = Rs 40.60, gain=16% is?
A. 30
B. 35
C. 40
D. 45
5. A person incurs loss for by selling a watch for rs1140 at what price should the watch be sold to earn a 5% profit?
A. 1260
B. 3300
C. 450
D. 50
6. By selling 33 metres of cloth, one gains the selling price of 11 metres. Then the gain percent is?
A. 50
B. 55
C. 60
D. 65
7. If the cost price is 96% of S.P. then what is the profit %?
A. 2.32
B. 3.67
C. 4.17
D. 5.98
8. A man brought toffees at for a rupee. How many for a rupee must he sell to gain 50%?
A. 1
B. 2
C. 4
D. 5
9. A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120kg sugar at Rs.16per kg. At what rate should he sell the mixer to gain 16%?
A. 30.2
B. 20.3
C. 17.4
D. 51.8
10. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960gms for a kg weight. Find his gain percent.

Analytical Skills

- A. 3.2
 - B. 4.1
 - C. 1.8
 - D. 5.8
11. Monika purchased a pressure cooker at $\frac{9}{10}$ th of its selling price and sold it at 8% more than its S. P. Then her gain percent is?
- A. 20
 - B. 12
 - C. 17
 - D. 4
12. If the manufacturer gains 10%, the wholesale dealer 15% and the retailer 25%, then find the cost of production of a, the retail price of which is Rs. 1265.
- A. 900
 - B. 800
 - C. 200
 - D. 300
13. A An article is sold at certain price. By selling it at $\frac{2}{3}$ of its price one losses 10%, find the gain at original price?
- A. 30
 - B. 20
 - C. 35
 - D. 51
14. A the single discount equivalent to a series discount of 20% ,10% and 5% is?
- A. 12
 - B. 76.2
 - C. 68.4
 - D. 55
15. A retailer buys 40 pens at the market price of 36 pens from a wholesaler, if he sells these pens giving a discount of 1%, what is the profit %?
- A. 30
 - B. 20
 - C. 15
 - D. 10

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. D | 2. D | 3. C | 4. B | 5. A |
| 6. B | 7. C | 8. B | 9. C | 10. B |
| 11. A | 12. B | 13. C | 14. C | 15. D |

Review Questions

- At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?
- An uneducated retailer marks all its goods at 50% above the cost price and thinking that he will still make 25% profit, offers a discount of 25% on the market price. what is the actual profit on the sales?
- A man bought a horse and a carriage for Rs 3000. he sold the horse at a gain of 20% and the carriage at a loss of 10%, thereby gaining 2% on the whole. find the cost of the horse.
- Alfred buys an old scooter for Rs. 4700 and spends Rs. 800 on its repairs. If he sells the scooter for Rs. 5800, his gain percent is?
- The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is?
- If selling price is doubled, the profit triples. Find the profit percent.
- In a certain store, the profit is 320% of the cost. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit?
- A vendor bought toffees at 6 for a rupee. How many for a rupee must he sell to gain 20%?
- The cost price of three varieties of oranges namely A, B and C is Rs 20/kg, Rs 40/kg and Rs 50/kg. Find the selling price of one kg of orange in which these three varieties of oranges are mixed in the ratio of 2 : 3 : 5 such that there is a net profit of 20%?
- A sweet seller sells $\frac{3}{5}$ th part of sweets at a profit of 10% and remaining at a loss of 5%. If the total profit is Rs 1500, then what is the total cost price of sweets?

**Further Reading**

- Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
- A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
- Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
- 4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

Unit 08: Simple Interest

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Summary

Keywords

Self Assessment

Answers for Self Assessment

Review Questions

Further Reading

Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Concept of principal, rate, time, simple interest, and amount.
- Explore the formulae of principal, rate, time, simple interest, and amount.
- Analyze effect of change of principal, rate, and time on simple interest.
- Solve problems based on simple interest computation, principal computation, rate, time, and amount computation.

Introduction

Problem related to computation of principal, rate, time, simple interest, and amounts asked in almost all competitive exams. In this chapter you will be exposed to the concept of principal, rate, time, simple interest, and amount; effect of change of principal, rate, and time on simple interest; and procedure to compute principal, rate, time, simple interest, and amount based on the given information.

8.1 Basics of Principal, Rate and Time

The basics of principal, rate, and time are explained below:

Principal

Principal is the total amount of money borrowed by an individual from other individual. Principal is the money borrowed or lent out for a certain period.

The principal is the amount that initially borrowed from the bank or invested.

Analytical Skills

It is generally denoted by 'P' or 'p'.

Rate/Rate of Interest

Interest is the extra money paid for using other's money. It is interest per cent. For example, 5% or 9%.

The interest is usually charged according to a specified term, which is expressed as some per cent of the principal and is called the rate of interest for a fixed period. It is generally denoted by 'R' or 'r'. Fixed period may be a year, six months, three months, a month, etc. Rate of interest may be calculated annually, semi-annually, quarterly, monthly etc. R % or r% in simple form are written as $\frac{R}{100}$ and $\frac{r}{100}$.



For example

- Rate of interest - 5%, 10%, or 12%, etc.
- Rate of interest is 5% per annum means Interest payable on 100 for one year is 5.

Time

It is defined as the duration for which the principal amount is given to someone. For example, a year, six months, three months, a month etc. It is generally denoted by 'T' and 't'.

Simple Interest

It is the interest on a sum borrowed for a certain period is reckoned uniformly. It is that interest which is payable on the principal amount only.

The general abbreviation of simple interest is 'S.I.' or 'I'.

It is the interest computed on the principal amount for the entire period it is borrowed.

For example, simple interest on Rs. 100 at 5% per annum = Rs. 5 each year.

Amount

Amount is defined as the principal borrowed plus the interest occurred on principal.

Formulae

- Simple Interest = $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$ or S.I. = $\frac{P \times R \times T}{100}$
- Principal = $\frac{\text{Simple Interest} \times 100}{\text{Rate} \times \text{Time}}$ or P = $\frac{\text{S.I.} \times 100}{R \times T}$
- Rate = $\frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Time}}$ or R = $\frac{\text{S.I.} \times 100}{P \times T}$
- Time = $\frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Rate}}$ or T = $\frac{\text{S.I.} \times 100}{P \times R}$

Amount = Principal + Simple Interest/Interest

or A = P (or p) + S.I. (or I)

Principal = Amount - Interest or P = A - S.I.

Interest = Amount - Principal or S.I. = A - P

Amount = Principal + $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

or A = $P + \frac{P \times R \times T}{100} = P \left(1 + \frac{RT}{100} \right)$

8.2 Computation of Simple Interest



Example 1

Calculate the simple interest on Rs. 3000 for 3 years at 5% per annum.

Solution: - Given,

$P = \text{Rs. } 3000$, $R = 5\%$ p.a., and $T = 3$ years

We know that $S.I. = \frac{P \times R \times T}{100}$

$$S.I. = \frac{3000 \times 5 \times 3}{100} = \text{Rs. } 450$$



Example 2

Find the simple interest on Rs. 32000 at $7\frac{1}{2}\%$ p.a. for 8 months.

Solution: - Given,

$$P = \text{Rs. } 32000, R = 7\frac{1}{2}\% \text{ p.a.} = \frac{15}{2}\% \text{ p.a.}, \& T = 8 \text{ months} = \frac{8}{12} \text{ Years} = \frac{2}{3} \text{ Years}$$

$$\text{We know that } S.I. = \frac{P \times R \times T}{100} = \frac{32000 \times 15 \times 2}{100 \times 2 \times 3} = \text{Rs. } 1600$$



Example 3

Find the simple interest on Rs 5000 at 15 % p.a. for the period from 25th May, 2022 to 28th July, 2022.

Solution: - Given,

$$P = \text{Rs. } 5000 \& R = 15\% \text{ p.a.}$$

$$T = (6 \text{ days of May} + 30 \text{ days of June} + 28 \text{ days of July})$$

$$T = 64 \text{ days} = \frac{64}{365} \text{ years}$$

$$\text{We know that } S.I. = \frac{P \times R \times T}{100} = \frac{5000 \times 15 \times 64}{100 \times 365}$$

$$S.I. = \text{Rs. } \frac{9600}{73}$$

$$S.I. = \text{Rs. } 131.51 \text{ (approx.)}$$



Example 4

The simple interest accrued on an amount of Rs. 1500 at the end of 2 years is Rs. 600. What would be the simple interest accrued on an amount of Rs. 3400 at the same rate and for the same period?

Solution: - Given,

$$P = \text{Rs. } 1500, T = 2 \text{ years}, \& S.I. = \text{Rs. } 600$$

$$\text{We know that } R = \frac{S.I. \times 100}{P \times T} = \frac{600 \times 100}{1500 \times 2} = 20\%$$

$$\text{So, } R = 20\% \text{ p.a. new Principal } P = \text{Rs. } 3400, \text{ and } T = 2 \text{ years}$$

$$\text{Therefore } S.I. = \frac{P \times R \times T}{100} = \frac{3400 \times 20 \times 2}{100} = \text{Rs. } 1360$$

8.3 Effect of Change of P, R & T on S.I.

Change in Simple Interest (S.I.) = [Product of Fixed Parameter] \times [Difference of Product of Variable Parameters]

100



Did you know?

- If P changes but R & T remains same, then change in S.I. = $\frac{[R \times T] \times [\text{Change in } P]}{100}$
- If R changes but P & T remains same, then change in S.I. = $\frac{[P \times T] \times [\text{Change in } R]}{100}$

Analytical Skills

- If T changes but P & R remains same, then change in S.I. = $\frac{[P \times R] \times [\text{Change in } T]}{100}$
- If R changes from R_1 to R_2 , T changes from T_1 to T_2 , and P is fixed, then
Change in S.I. = $\frac{P \times [R_1 \times T_1 - R_2 \times T_2]}{100}$
- If P changes from P_1 to P_2 , T changes from T_1 to T_2 , and R is fixed, then
Change in S.I. = $\frac{R \times [P_1 \times T_1 - P_2 \times T_2]}{100}$
- If P changes from P_1 to P_2 , R changes from R_1 to R_2 , and T is fixed, then
Change in S.I. = $\frac{T \times [P_1 \times R_1 - P_2 \times R_2]}{100}$

**Example 5**

Calculate the change in simple interest (S.I.) for 3 years at 2% p.a. if the principal increases by Rs.1000.

Solution: - Given,

Change in P = Rs. 1000, R = 2 % p.a., & T = 3 years

We know that Change in S.I. = $\frac{[R \times T] \times [\text{Change in } P]}{100} = \frac{2 \times 3 \times 1000}{100} = \text{Rs. } 60$

**Example 6**

Calculate the change in simple interest (S.I.) on Rs. 500 for 3 years if rate % increases by 2% p.a.

Solution: - Given,

P = Rs. 500, Change in R = 2 % p.a., & T = 3 years

We know that Change in S.I. = $\frac{[P \times T] \times [\text{Change in } R]}{100} = \frac{500 \times 3 \times 2}{100} = \text{Rs. } 30$

**Example 7**

Calculate the change in simple interest (S.I.) on Rs. 500 at 2% p.a. if the time increases by 6 years.

Solution: - Given,

P = Rs. 500, R = 2 % p.a., & Change in T = 6 years

We know that Change in S.I. = $\frac{[P \times R] \times [\text{Change in } T]}{100} = \frac{500 \times 2 \times 6}{100} = \text{Rs. } 60$

**Example 8**

Calculate the change in simple interest (S.I.) on Rs. 8000 if the rate % changes from 4% p.a. to 5% p.a. and the time changes from 4 years to 3 years.

Solution: - Given,

P = Rs. 8000, $R_1 = 4$ % p.a., $R_2 = 5$ % p.a., $T_1 = 4$ yrs. & $T_2 = 3$ yrs.

We know that Change in S.I. = $\frac{P \times [R_1 \times T_1 - R_2 \times T_2]}{100} = \frac{8000 \times [4 \times 4 - 5 \times 3]}{100} = 80 \times 1 = \text{Rs. } 80$

8.4 Computation of Principal



Example 9

A man earns Rs. 450 as interest in 3 years on a certain money invested in a company at the rate of 5% p.a. Calculate the principal invested by the man in the company.

Solution: - Given,

S.I. = Rs. 450, R = 5% p.a., and T = 3 years

P = ?

We know that $P = \frac{S.I. \times 100}{R \times T} = \frac{450 \times 100}{5 \times 3} = \text{Rs. } 3000$

P = Rs. 3000



Example 10

What principal will amount to Rs. 2500 at 2% p.a. in $12\frac{1}{2}$ years.

Solution: - Given,

A = Rs. 2500, R = 2% p.a., and T = $12\frac{1}{2}$ years = $\frac{25}{2}$ years

P = ?

We know that $P = \frac{A \times 100}{100 + R \times T} = \frac{2500 \times 100}{100 + 2 \times \frac{25}{2}} = \frac{250000}{125} = \text{Rs. } 2000$



Example 11

Calculate the annual instalment that will discharge a debt of Rs.12900 due in 4 years at 5% p.a. simple interest.

Solution: - Given

A = Rs. 6500, R = 3% p.a., and T = 5 years

Let annual instalment is X

Annual instalment (X) = ?

We know that $A = P + \frac{P \times R \times T}{100} = P \left(1 + \frac{RT}{100}\right) = A_1 + A_2 + A_3 + A_4 + A_5 = X \left(1 + \frac{5 \times 3}{100}\right) + X \left(1 + \frac{5 \times 2}{100}\right) + X \left(1 + \frac{5 \times 1}{100}\right) + X \left(1 + \frac{5 \times 0}{100}\right) = 12900$

Or, $X \left(4 + \frac{15+10+5+0}{100}\right) = 12900$

X = Rs. 3000

Therefore, annual instalment = Rs. 3000



Example 12

Calculate principal if it amounts to Rs. 5000 in 2 years and to Rs. 6000 in 3 years at simple interest.

Solution: - Given,

$A_1 = \text{Rs. } 5000$, $A_2 = \text{Rs. } 6000$, $T_1 = 2$ years, and $T_2 = 3$ years

P = ?

Analytical Skills

We know that $A = P + \frac{P \times R \times T}{100} = P \left(1 + \frac{RT}{100} \right)$

$$A_1 = \text{Rs. } 5000 = P \left(1 + \frac{2R}{100} \right)$$

$$A_2 = \text{Rs. } 6000 = P \left(1 + \frac{3R}{100} \right)$$

$$\frac{A_2}{A_1} = 1.2 = \frac{1 + 0.03R}{1 + 0.02R}$$

$$\text{Or, } 1.2 + 0.024R = 1 + 0.03R$$

$$\text{Or, } 0.006R = 0.2$$

$$R = 33.333\%$$

$$\text{Or, } P = \frac{5000}{1 + \frac{2 \times 33.33}{100}} = \text{Rs. } 3000$$

Alternatively,

$$P = \frac{A_1 \times T_2 - A_2 \times T_1}{T_2 - T_1} = \frac{[5000 \times 3 - 6000 \times 2]}{3 - 2} = \text{Rs. } 3000$$

**Example 13**

Calculate the sum if it amounts to Rs. 500 at 8% p.a. and amounts to Rs. 400 at 4% p.a.

Solution: - Given,

$$A_1 = \text{Rs. } 500, A_2 = \text{Rs. } 400, R_1 = 8\% \text{ p.a., and } R_2 = 4\% \text{ p.a. } P = ?$$

$$\text{We know that } P = \frac{A_1 \times R_2 - A_2 \times R_1}{R_2 - R_1} = (500 \times 4 - 400 \times 8) / (4 - 8) = \text{Rs. } 300$$

**Example 14**

What will be the original sum of money if annual income is Rs. 2400, $\frac{1}{2}$ of it is invested at 1%, $\frac{1}{4}$ at 3% and the rest at 5%.

Solution: - Given,

$$\text{S.I.} = 2400, R_1 = 1\% \text{ p.a., } R_2 = 3\% \text{ p.a., and } R_3 = 5\% \text{ p.a. assume total investment is } P. \text{ Therefore, } P_1 = \frac{P}{2}, P_2 = \frac{P}{4}, \text{ and Rest, } P_3 = P - \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$$

$$\text{Here, S.I.} = S.I._1 + S.I._2 + S.I._3 = \frac{P \times 1 \times 1}{200} + \frac{P \times 3 \times 1}{400} + \frac{P \times 5 \times 1}{400} = \frac{P}{40} = 2400$$

$$\text{Or, } P = \text{Rs. } 96000$$

**Example 15**

Calculate the sum if the simple interest on a certain sum of money for 6 years at 10% p.a. is Rs. 30 less than the simple interest on the same sum for 5 years at 8% p.a.

Solution: - Given,

$$\text{Difference in S.I.} = \text{Rs. } 30, R_1 = 10\% \text{ p.a., } T_1 = 6 \text{ years, } T_2 = 5 \text{ years, and } R_2 = 8\% \text{ p.a.}$$

$$P = ?$$

$$\text{We know that S.I.} = \frac{P \times R \times T}{100}$$

$$\text{Difference in S.I.} = \frac{P}{100} (R_1 \times T_1 - R_2 \times T_2) = \frac{P \times (10 \times 6 - 8 \times 5)}{100} = \frac{P}{5} = 30$$

Or, $P = \text{Rs. } 150$

8.5 Computation of Rate



Example 16

At what interest rate p.a., the simple interest accrued on an amount of Rs. 1500 at the end of 2 years is Rs. 600?

Solution - Given,

$P = \text{Rs. } 1500$, $T = 2$ years, and $\text{S.I.} = \text{Rs. } 600$

$R = ?$

We know that $R = \frac{\text{S.I.} \times 100}{P \times T} = \frac{600 \times 100}{1500 \times 2} = 20\%$



Example 17

At what interest rate p.a., in 4 years, a sum of Rs. 2000 will become Rs. 4000?

Solution: - Given,

$P = \text{Rs. } 2000$, $A = \text{Rs. } 4000$, $T = 4$ years

$R = ?$ and $\text{S.I.} = ?$

We know that $\text{S.I.} = A - P = \text{Rs. } (4000 - 2000) = \text{Rs. } 2000$

We know that $R = \frac{\text{S.I.} \times 100}{P \times T} = \frac{2000 \times 100}{2000 \times 4} = 25\% \text{ p.a.}$



Example 18

Calculate rate per cent p.a. if a sum of money trebles (increase three times) itself in 4 years simple interest.

Solution: - Given, $A = 3P$

We know that $\text{S.I.} = A - P = 3P - P = 2P$

$$\text{S. I.} = \frac{P \times R \times T}{100} = \frac{P \times R \times 4}{100} = 2P$$

$$\text{Or, } R = \frac{2 \times 100}{4} = 50\% \text{ p.a.}$$



Example 19

Calculate rate of interest if a sum of money at simple interest amounts to Rs. 5000 in 2 years and to Rs. 6000 in 3 years.

Solution: - Given,

$A_1 = \text{Rs. } 5000$, $A_2 = \text{Rs. } 6000$, $T_1 = 2$ years, $T_2 = 3$ years, and $R = ?$

We know that $A_1 = P \left(1 + \frac{RT_1}{100} \right)$

$$A_2 = P \left(1 + \frac{RT_2}{100} \right)$$

$$\text{Or, } \frac{A_1}{A_2} = \frac{\left(1 + \frac{RT_1}{100} \right)}{\left(1 + \frac{RT_2}{100} \right)} = \frac{1 + 0.02R}{1 + 0.03R} = \frac{5000}{6000} = \frac{5}{6}$$

$$\text{Or, } 6 + 0.12R = 5 + 0.15R$$

Analytical Skills

$$\text{Or, } 0.03R=1 \text{ Or, } R=\frac{1}{0.03}=\frac{100}{3}=33\frac{1}{3}\%$$

Alternatively

$$R=\frac{A_1-A_2}{A_2T_1-A_1T_2}\times 100=\frac{5000-6000}{6000\times 2-5000\times 3}\times 100=\frac{-1000}{-3000}\times 100=\frac{100}{3}=33\frac{1}{3}\%$$

**Example 20**

A person 'X' deposits Rs. 1000 and Rs. 500 in saving at 3.5% p.a. and 5% p.a. respectively. Compute the rate of interest for the whole sum.

Solution: - Given,

P = Rs. 1000, P = Rs. 500, R = 3.5% p.a., and R = 5% p.a. R = ?

$$\text{We know that } R=\frac{P_1\times R_1+P_2\times R_2}{P_1+P_2}=\frac{[1000\times 3.5+500\times 5]}{[1000+500]}=\frac{3500+1500}{1500}=\frac{6000}{1500}=4\% \text{ p.a.}$$

8.6 Computation of Time**Example 21**

In what time Rs.20000 will earn an interest of Rs.2000 at 8% p.a.?

Solution: - Given,

P = Rs. 20000, S.I. = Rs. 2000, and R = 8% p.a.

T = ?

$$\text{We know that } T=\frac{S.I.\times 100}{P\times R}=\frac{2000\times 100}{20000\times 8}=\frac{10}{8}=1\frac{1}{4}\text{Years}=1\text{ year and 3 months}$$

**Example 22**

In what time a sum of money will four times itself at a rate of simple interest of 12% p.a.?

Solution: - Given, R=12%, A =4P, and T=?

We know that S.I. = A - P=4P-P=3P

$$\text{S. I.}=\frac{P\times R\times T}{100}=\frac{P\times 12\times T}{100}=3P$$

$$\text{Or, } T=\frac{3\times 100}{12}=25\text{ Years}$$

**Example 23**

In how many years will a sum of money double itself at $13\frac{1}{2}\%$ p.a.?

Solution: - Given, $R=13\frac{1}{2}\%=\frac{27}{2}\%$, A =2P, and T=?

We know that S.I. = A - P=2P-P=P

$$\text{S. I.}=\frac{P\times R\times T}{100}=\frac{P\times 27\times T}{200}=P$$

$$\text{Or, } T=\frac{200}{27}=7\frac{11}{27}\text{ Years}$$

**Example 24**

A sum of money put out on simple interest doubles itself in 10 years. In how many years would it five times itself?

Solution: - Given, $A = 2P$, and $T = 10$ Years

We know that $S.I. = A - P = 2P - P = P$

$$S.I. = \frac{P \times R \times T}{100} = P$$

$$R = 10\%$$

NOW, $A = 5P$, Or, $S.I. = A - P = 4P$

$$S.I. = \frac{P \times 10 \times T}{100} = 4P$$

Or, $T = 40$ years

Alternatively,

In given question only time is variable other two parameters (principal and rate of interest) is constant.

Therefore, we can write

$$S.I. \propto T$$

In first Case $S.I. = A - P = 2P - P = P$

$$P \propto 10 \text{ Years}$$

In second Case

$A = 5P$, Or, $S.I. = A - P = 4P$

$$4P \propto 40 \text{ Years}$$

Therefore, Required Time (T') = 40 Years



Example 25

A certain sum of money becomes three times of itself in 20 years at simple interest. In how many years does it become double of itself at the same rate of simple interest?

Solution:

In given question only time is variable other two parameters (principal and rate of interest) is constant.

Therefore, we can write

$$S.I. \propto T$$

In first Case $S.I. = A - P = 3P - P = 2P$

$$2P \propto 20 \text{ Years Or, } P \propto 10 \text{ Years}$$

In second Case

$A = 2P$, Or, $S.I. = A - P = P$

$$P \propto 10 \text{ Years}$$

Therefore, Required Time (T') = 10 Years



Example 26

Calculate the time if a sum amounts to Rs. 500 at 8% p.a. and amounts to Rs. 400 at 4% p.a.

Solution: - Given,

$$A_1 = \text{Rs. } 500, A_2 = \text{Rs. } 400, R_1 = 8\% \text{ p.a. and } R_2 = 4\% \text{ p.a. } T = ?$$

$$\text{We know that } A_1 = P \left(1 + \frac{R_1 T}{100} \right)$$

$$A_2 = P \left(1 + \frac{R_2 T}{100} \right)$$

Analytical Skills

$$\text{Or, } \frac{A_1}{A_2} = \frac{\left(1 + \frac{R_1 T}{100}\right)}{\left(1 + \frac{R_2 T}{100}\right)} = \frac{1 + 0.08T}{1 + 0.04T} = \frac{500}{400} = \frac{5}{4}$$

$$\text{Or, } 4 + 0.32T = 5 + 0.20T$$

$$\text{Or, } 0.12T = 1 \text{ Or, } T = \frac{1}{0.12} = \frac{100}{12} = 8\frac{1}{3} \text{ Years}$$

Alternatively

$$T = \frac{A_1 - A_2}{A_2 R_1 - A_1 R_2} \times 100 = \frac{500 - 400}{400 \times 8 - 500 \times 4} \times 100 = \frac{100}{1200} \times 100 = \frac{100}{12} = 8\frac{1}{3} \text{ Years}$$

8.7 Computation of Amount**Example27**

If the interest rate is 3% p.a., then a sum of Rs. 2000 amount to how much in 4 years?

Solution: - Given,

P = Rs. 2000, R = 3% p.a., and T = 4 years

A = ?

$$\text{We know that S.I.} = \frac{P \times R \times T}{100} = \frac{2000 \times 3 \times 4}{100} = \text{Rs. } 240$$

$$A = P + \text{S.I.}$$

$$A = \text{Rs. } (2000 + 240)$$

$$A = \text{Rs. } 2240$$

**Example28**

A person borrowed Rs. 7000 from his friend at 5% p.a. for 2 years. Find the money returned by that person to his friend.

Solution: - Given,

P = Rs. 7000, R = 5% p.a., and T = 2 years

A = ?

$$\text{We know that } A = P \left(1 + \frac{R \times T}{100}\right) = 7000 \left(1 + \frac{5 \times 2}{100}\right) = 7000 \times 1.1 = \text{Rs. } 7700$$

Summary

The key concepts learned from this unit are: -

- In short, Simple Interest = $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$ or S.I. = $\frac{P \times R \times T}{100}$
- Principal = $\frac{\text{Simple Interest} \times 100}{\text{Rate} \times \text{Time}}$ or $P = \frac{\text{S.I.} \times 100}{R \times T}$
- Rate = $\frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Time}}$ or $R = \frac{\text{S.I.} \times 100}{P \times T}$
- Time = $\frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Rate}}$ or $T = \frac{\text{S.I.} \times 100}{P \times R}$
- Amount = Principal + Simple Interest/Interest
- or $A = P \text{ (or p) } + \text{S.I. (or I)}$
- Principal = Amount - Interest or $P = A - \text{S.I.}$
- Interest = Amount - Principal or $\text{S.I.} = A - P$
- Amount = Principal + $\frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$
- or $A = P + \frac{P \times R \times T}{100} = P \left(1 + \frac{RT}{100}\right)$

Keywords

- Principal; Rate; Time; Simple Interest; Amount.

Self Assessment

1. The simple interest on Rs. 500 at 6% per annum from May 3rd to July 15th in the same year is:
A. Rs. 9
B. Rs. 6
C. Rs. 4
D. Rs. 5
2. What would be the simple interest obtained on an amount of ` 5760 at the rate of 6% p.a. after 3 years?
A. Rs. 1036.80
B. Rs. 1063.80
C. Rs. 1336.80
D. Rs. 1666.80
3. A man borrowed a sum of Rs. 10000 from a finance company for 6 years at 8% per annum. The amount returned by man to the finance company is
A. Rs. 14800
B. Rs. 12600
C. Rs. 13300
D. Rs. 12040
4. A farmer borrowed Rs. 3600 at 15% simple interest per annum. At the end of 4 years, he cleared this account by paying Rs. 4000 and a cow. The cost of the cow is
A. Rs. 1000
B. Rs. 1200
C. Rs. 1550
D. Rs.1760
5. The principal that will yield Rs. 60 as simple interest at 6% per annum in 5 years is
A. Rs. 175
B. Rs. 350
C. Rs. 200
D. Rs. 259
6. 'X' borrows Rs. 520 from 'Y' at a simple interest of 13% per annum. What amount of money should 'X' pay to 'Y' after 6 months to be absolved of the debt?
A. Rs. 353.80
B. Rs. 453.80
C. Rs.552.80
D. Rs. 553.80

Analytical Skills

7. The sum of money that will produce Rs. 1770 interest in 7 years at 8% simple interest per annum is
- A. Rs. 2950
 - B. Rs. 3120
 - C. Rs. 2800
 - D. Rs. 1359
8. A sum fetched a total simple interest of Rs. 4016.25 at the rate of 9 p.c. p.a. in 5 years. What is the sum?
- A. Rs. 4462.50
 - B. Rs. 8032.50
 - C. Rs. 8900
 - D. Rs. 8925
9. If the simple interest on a certain sum of money after 6 years is of the principal, then the rate of interest per annum is
- A. 5%
 - B. 6%
 - C. 4%
 - D. 7%
10. In 4 years, Rs. 6000 amounts to Rs. 8000. In what time at the same rate will Rs. 525 amount to Rs. 700?
- A. 2 years
 - B. 3 years
 - C. 4 years
 - D. 5 years
11. In how many years will a sum of money treble itself at 10% per annum simple interest?
- A. 15 years
 - B. 19 years
 - C. 20 years
 - D. 12 years
12. Rs. 6200 amounts to Rs. 9176 in 4 years at simple interest. If the interest rate is increased by 3% it would amount to how much?
- A. Rs. 8432
 - B. Rs. 9820
 - C. Rs. 9920
 - D. Rs. 10920
13. A sum of money doubles itself in 8 years. In how many years will it treble?
- A. 16 years
 - B. 15 years
 - C. 14 years
 - D. 21 years

14. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2% higher rate, it would have fetched Rs. 56 more. Find the sum.
- A. Rs. 680
B. Rs. 700
C. Rs. 720
D. Rs. 530
15. A certain amount earns simple interest of Rs. 1750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?
- A. Rs. 35
B. Rs. 245
C. Rs. 350
D. Cannot be determined

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. A | 3. A | 4. D | 5. C |
| 6. D | 7. A | 8. D | 9. B | 10. C |
| 11. C | 12. C | 13. A | 14. D | 15. B |

Review Questions

1. If the simple interest on Rs. 1400 be more than the interest on Rs. 1000 by Rs. 60 in 5 years, then find the out the rate per cent per annum.
2. A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But, when invested at 5% per annum, it amounts to `350. Find out the sum and time.
3. What is the present worth of Rs.132 due in 2 years at 5% simple interest per annum?
4. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year?
5. Out of a certain sum, is invested at 3%, at 6% and the rest at 8%. If the annual income is Rs. 300, then find original sum.
6. In how much time would the simple interest on a certain sum be 0.125 times the principal at 10% per annum?
7. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is Rs. 840. At what rate of interest the same amount of interest can be received on the same sum after 5 years?
8. If the simple interest on `1400 be more than the interest on `1000 by `60 in 5 years, find the out the rate per cent per annum.
9. A sum of money put out on simple interest doubles itself in 12 years. In how many years would it treble itself?
10. Rahul deposits Rs. 5000 in NSC at 2% per annum and Rs. 2000 in mutual funds at 4% per annum. Find out the rate of interest for the whole sum.

**Further Reading**

- Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing. 2. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal. S. Chand & Co Ltd. (2010).
- Magical Book on Quicker Maths by M Tyra, Banking Service Chronicle.
- Analytical Reasoning by M.K. Pandey, Banking Service Chronicle.
- Quantitative Aptitude for Competitive Examinations by Dinesh Khattar, Pearson Education (2020).

**Web Links**

- <https://www.hitbullseye.com/quant>
- [https://www.indiabix.com/aptitude/questions-and- answers/](https://www.indiabix.com/aptitude/questions-and-answers/)
- 3. <https://www.examveda.com/mcq-question-on- arithmetic-ability/>

Unit 09: Compound Interest

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9.6 Relation between Compound and Simple Interest

Summary

Keywords

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Concept of compound interest.
- Explore the formulae of compound interest.
- Differentiate between simple and compound interest.
- Solve problems based on compound interest computation.
- Solve problems based on principal and rate computation.
- Solve problems based on time and amount computation.
- Solve problems based on relation between compound and simple interest.

Introduction

Problem related to computation of principal, rate, time, compound interest, and amount is asked in almost all competitive exams. In this chapter you will be exposed to the concept of compound interest; formulae of compound interest; and procedure to solve problems based on compound interest, principal, rate, time, amount, and relation between compound and simple interest for given data.

9.1 Compound Interest

When the borrower and lender agree to fix up a specific unit of time i.e., yearly/half-yearly/quarterly to settle the previous account, then the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

Analytical Skills

In this situation, the difference between the money borrowed and amount is called the compound interest for that specific period. The general abbreviation used for compound interest is C.I.

Mathematically, Compound Interest = Amount - Principal

Basic Formulae

Let P or p = Principal, R or r = Rate, N or n = Number of years (T or t), C.I. = Compound interest

A = Amount

Amount in CI

- Compound Interest = Amount - Principals
- When interest is compounded annually
- $P + C.I. = A$
- $P + \frac{PR}{100} = P \left(1 + \frac{R}{100}\right) = A_1$ (Amount after first year)
- $P \left(1 + \frac{R}{100}\right) + P \left(1 + \frac{R}{100}\right) \frac{R}{100} = P \left(1 + \frac{R}{100}\right)^2 = A_2$ (Amount after first year)
- $P \left(1 + \frac{R}{100}\right)^2 + P \left(1 + \frac{R}{100}\right)^2 \frac{R}{100} = P \left(1 + \frac{R}{100}\right)^3 = A_3$ (Amount after third year)

$$A_n = P \left(1 + \frac{R}{100}\right)^n \text{ (Amount after 'n' year)}$$

$$\text{Compound Interest (C.I.)} = P \left(1 + \frac{R}{100}\right)^n - P = P \left[\left(1 + \frac{R}{100}\right)^n - 1\right]$$

Rate of interest

$$A = P \left(1 + \frac{R}{100}\right)^n \text{ Or, } \frac{A}{P} = \left(1 + \frac{R}{100}\right)^n$$

$$\left(\frac{A}{P}\right)^{1/n} - 1 = \frac{R}{100}$$

$$\text{Rate of interest (R)} = \left[\left(\frac{A}{P}\right)^{1/n} - 1\right] \times 100\%$$

When interest is compounded Half-yearly

$$\text{Then Rate of interest for half year } R_H = \frac{R}{2}$$

$$\text{And time for half year } n_H = 2n$$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n}$$

$$\text{Compound Interest} = P \left[\left(1 + \frac{R/2}{100}\right)^{2n} - 1\right]$$

$$\text{Rate of interest (R)} = 2 \times 100 \times \left[\left(\frac{A}{P}\right)^{1/2n} - 1\right]\%$$

When interest is compounded Quarterly-yearly

$$\text{Then Rate of interest for quarter year } R_Q = \frac{R}{4}$$

$$\text{And time for quarter year } n_Q = 4n$$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/4}{100} \right)^{4n}$$

$$\text{Compound Interest} = P \left[\left(1 + \frac{R/4}{100} \right)^{4n} - 1 \right]$$

$$\text{Rate of interest (R)} = 4 \times 100 \times \left[\left(\frac{A}{P} \right)^{1/4n} - 1 \right] \%$$

When interest is compounded annually but time is in fraction, $1\frac{1}{2}$ years.

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^1 \times \left(1 + \frac{R/2}{100} \right)$$

$$\text{Compound Interest} = P \left[\left(1 + \frac{R}{100} \right)^1 \times \left(1 + \frac{R/2}{100} \right) - 1 \right]$$

When rates are different for different years

$R_1\%, R_2\%, R_3\%, R_4\%, \dots$ for 1st, 2nd, 3rd, 4th year respectively.

$$\text{Amount} = P \times \left(1 + \frac{R_1}{100} \right) \times \left(1 + \frac{R_2}{100} \right) \times \left(1 + \frac{R_3}{100} \right) \times \left(1 + \frac{R_4}{100} \right) \times \dots$$

Present worth of Rs. x due n years hence

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100} \right)^n}$$

Difference between CI & SI:

For First years:

$$CI - SI = 0$$

Derivation

$$\text{Simple interest on Rs. P for 1 years at R\% per annum} = \frac{P \times R \times 1}{100}$$

$$\text{Compound interest on Rs. P for 1 years at R\% per annum} = P \left(1 + \frac{R}{100} \right)^1 - P$$

$$C.I. - S.I. = P \left(1 + \frac{R}{100} \right)^1 - P - \frac{P \times R \times 1}{100} = P + \frac{P \times R}{100} - P - \frac{P \times R \times 1}{100} = 0$$

$$\text{Or, } (C.I. - S.I.)_{1 \text{ years}} = 0$$

The difference between the compound interest and the simple interest on a certain sum of money for 2 years at R% per annum

In terms of P and R

$$CI - SI = P \left(\frac{R}{100} \right)^2$$

In terms of S.I. and R

$$\bullet \text{ C.I.} - \text{S.I.} = \frac{R \times S.I.}{2 \times 100}$$

Derivation- Let, given sum of money = Rs. P

$$\text{Simple interest on Rs. P for 2 years at R\% per annum} = \frac{P \times R \times 2}{100}$$

Compound interest on Rs. P for 2 years at R% per annum = $P \left(1 + \frac{R}{100}\right)^2 - P$

$$C.I. - S.I. = P \left(1 + \frac{R}{100}\right)^2 - P - \frac{P \times R \times 2}{100} = P + \frac{2 \times P \times R}{100} + P \times \left(\frac{R}{100}\right)^2 - P - \frac{P \times R \times 2}{100} = P \times \left(\frac{R}{100}\right)^2$$

$$\text{Or, } (C.I. - S.I.)_{2 \text{ years}} = P \left(\frac{R}{100}\right)^2 = P \times \frac{R}{100} \times \frac{R}{100} = \frac{P \times R \times 2}{100} \times \frac{R}{2 \times 100} = \frac{S.I. \times R}{2 \times 100}$$

The difference between the compound interest and the simple interest on a certain sum of money for 3 years at R% per annum

In terms of P and R

$$(C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100}\right)^3 + 3P \left(\frac{R}{100}\right)^2$$

$$\text{Or } (C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100}\right)^2 \left(\frac{R}{100} + 3\right)$$

In terms of S.I. and R

$$(C.I. - S.I.)_{3 \text{ years}} = \frac{S.I.}{3} \left[\left(\frac{R}{100}\right)^2 + 3 \left(\frac{R}{100}\right) \right]$$

Derivation- Let, given sum of money = Rs. P

Simple interest on Rs. P for 3 years at R% per annum $\frac{P \times R \times 3}{100}$

Compound interest on Rs. P for 3 years at R% per annum = $P \left(1 + \frac{R}{100}\right)^3 - P$

$$(C.I. - S.I.)_{3 \text{ years}} = P \left(1 + \frac{R}{100}\right)^3 - P - \frac{P \times R \times 3}{100} = P + \frac{3 \times P \times R}{100} + 3P \times \left(\frac{R}{100}\right)^2 + P \times \left(\frac{R}{100}\right)^3 - P - \frac{P \times R \times 3}{100} = P \left(\frac{R}{100}\right)^3 + 3P \left(\frac{R}{100}\right)^2$$

$$\text{Or } (C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100}\right)^2 \left(\frac{R}{100} + 3\right) = P \times \frac{R}{100} \times \frac{R}{100} \times \left(\frac{R}{100} + 3\right) = \frac{P \times R \times 3}{100} \times \frac{R}{3 \times 100} \times \left(\frac{R}{100} + 3\right) = \frac{S.I.}{3} \times \left[\left(\frac{R}{100}\right)^2 + 3 \frac{R}{100} \right]$$

$$\text{Or, } (C.I. - S.I.)_{3 \text{ years}} = \frac{S.I.}{3} \times \left[\left(\frac{R}{100}\right)^2 + 3 \frac{R}{100} \right]$$

If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Derivation- Let the sum of money = Rs. P

$$n \times P = P \left(1 + \frac{R}{100}\right)^t$$

$$\text{Or, } n = \left(1 + \frac{R}{100}\right)^t \dots \dots \dots (1)$$

Let the sum become n^m times in T years.

$$n^m = \left(1 + \frac{R}{100}\right)^T = \left(1 + \frac{R}{100}\right)^{mt}$$

T = mt years

Hence, sum becomes n^m times in mt years.

If a loan of Rs. P at R% compound interest per annum is to be repaid in 'n' equal yearly instalments, then the value of each instalment is given by per annum is

$$\frac{P}{\left(\frac{100}{100+R}\right)^1 + \left(\frac{100}{100+R}\right)^2 + \left(\frac{100}{100+R}\right)^3 + \dots + \left(\frac{100}{100+R}\right)^n}$$

Derivation: - Principal for Rs. X due at end of first year at R% = $\frac{100X}{100+R}$

Principal for Rs. X due at end of second year at R% = $\left(\frac{100}{100+R}\right)^2 X$

.....

Principal for Rs. X due at end of nth year at R% = $\left(\frac{100}{100+R}\right)^n X$

$$\left(\frac{100}{100+R}\right)^1 X + \left(\frac{100}{100+R}\right)^2 X + \left(\frac{100}{100+R}\right)^3 X + \dots + \left(\frac{100}{100+R}\right)^n X = P$$

$$X = \frac{P}{\left(\frac{100}{100+R}\right)^1 + \left(\frac{100}{100+R}\right)^2 + \left(\frac{100}{100+R}\right)^3 + \dots + \left(\frac{100}{100+R}\right)^n}$$

9.2 Computation of Compound Interest



Example 1

How much compound interest will be obtained on Rs.8,000 at the interest rate of 5 % p.a. after 2 years?

Solution: - Given,

P = Rs. 8,000; R = 5 % p.a.; and n = 2 years

A =? and C.I. =?

We know that $A = P \left(1 + \frac{R}{100}\right)^n = 8000 \left(1 + \frac{5}{100}\right)^2 = 8000 \times \left(\frac{21}{20}\right)^2 = 20 \times 441 = \text{Rs. } 8,820$

Also, C.I. = A - P

C.I. = 8,820 - 8,000 = Rs. 820



Example 2

Find compound interest on Rs. 3,000 at 5% p.a. for 2 years compounded annually.

Solution - Given,

P = Rs. 3,000; R = 5% p.a.; and n = 2 years

C.I. =?

We know that $A = P \left(1 + \frac{R}{100}\right)^n = 3000 \left(1 + \frac{5}{100}\right)^2 = 3000 \times \left(\frac{21}{20}\right)^2 = 7.5 \times 441 = \text{Rs. } 3,307.5$

Analytical Skills

Also, C.I. = A - P

$$\text{C.I.} = 3,307.5 - 3,000 = \text{Rs. } 307.5$$

$$\text{C.I.} = \text{Rs. } 307.5$$

**Example 3**

Find the compound interest on Rs. 4,000 at 10% p.a. for 2 years 6 months, compounded annually.

Solution - Given,

$$P = \text{Rs. } 4,000; R = 10 \% \text{ p.a.}; n = 2 \text{ years } 6 \text{ months} = 2\frac{6}{12} \text{ Years} = 2\frac{1}{2} \text{ Years}$$

$$\begin{aligned} \text{We know that Compound Interest for fraction } 2\frac{1}{2} \text{ Years} &= P \left[\left(1 + \frac{R}{100}\right)^2 \times \left(1 + \frac{R/2}{100}\right) - 1 \right] = \\ 4000 \left[\left(1 + \frac{10}{100}\right)^2 \times \left(1 + \frac{10/2}{100}\right) - 1 \right] &= 4000[1.1^2 \times 1.05 - 1] = \text{Rs. } 1082 \end{aligned}$$

**Example 4**

Find compound interest on Rs. 5,000 at 4% p.a. for 2 years compounded half-yearly.

Solution: - Given,

$$P = \text{Rs. } 5,000; R = 4 \% \text{ p.a.};$$

When interest is compounded Half-yearly

$$\text{Then Rate of interest for half year } R_H = \frac{R}{2}$$

$$\text{And time for half year } n_H = 2n$$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n} = 5000 \left(1 + \frac{4/2}{100}\right)^{2 \times 2} = 5000 \times 1.02^4 = \text{Rs. } 5412.16$$

Also, C.I. = A - P

$$\text{C.I.} = 5,412.16 - 5,000 = \text{Rs. } 412.16$$

$$\text{C.I.} = \text{Rs. } 412.16$$

**Example 5**

Compute the compound interest on Rs. 3,000 at 20% p.a. compounded quarterly for 1 year.

Solution: - Given,

$$P = \text{Rs. } 3,000; R = 20 \% \text{ p.a. compounded quarterly; and } n = 1 \text{ year (4 quarters)}$$

$$\text{C.I.} = ?$$

When interest is compounded Quarterly

$$\text{Then Rate of interest for Quarterly } R_Q = \frac{R}{4}$$

And time for Quarter year $n_Q = 4n$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/4}{100}\right)^{4n} = 3000 \left(1 + \frac{20/4}{100}\right)^{4 \times 1} = 3000 \times 1.05^4 = \text{Rs. } 3646.5$$

Also, C.I. = A - P

$$\text{C.I.} = 3,646.5 - 3,000 = \text{Rs. } 646.5$$

$$\text{C.I.} = \text{Rs. } 646.5$$



Example 6

Compute the compound interest on Rs. 12000 at 16% p.a. for 9 months, compounded quarterly.

Solution: -

$P = \text{Rs. } 12000$; $n = 9 \text{ months} = \frac{3}{4} \text{ Years} = 3 \text{ quarters}$; and $R = 16\% \text{ p.a. compounded quarterly}$

C.I. = ?

We know that Amount for quarterly compounding

$$= P \left(1 + \frac{R/4}{100}\right)^{4n} = 12000 \left(1 + \frac{16/4}{100}\right)^3 = 12000 \times 1.04^3 = \text{Rs. } 13,498.4$$

Also, C.I. = A - P

$$\text{C.I.} = 13,498.4 - 12,000 = \text{Rs. } 1,498.3$$

$$\text{C.I.} = \text{Rs. } 1,498.3$$



Example 7

Harish invests Rs. 5,000 in a bond which gives interest at 2% p.a. during the first year, 5% p.a. during the second year and 10% p.a. during the third year. Calculate compound interest.

Solution: - Given,

$P = \text{Rs. } 5,000$; $R = 2\% \text{ p.a.}$; $R = 5\% \text{ p.a.}$; and $R = 10\% \text{ p.a.}$

C.I. = ?

We know that when rate of interest different then Amount

$$\begin{aligned} &= P \times \left(1 + \frac{R_1}{100}\right) \times \left(1 + \frac{R_2}{100}\right) \times \left(1 + \frac{R_3}{100}\right) \times \dots \dots \dots \\ &= 5000 \times \left(1 + \frac{2}{100}\right) \times \left(1 + \frac{5}{100}\right) \times \left(1 + \frac{10}{100}\right) \\ &= 5000 \times 1.02 \times 1.05 \times 1.1 = \text{Rs. } 5890.5 \end{aligned}$$

Also, C.I. = A - P

$$\text{C.I.} = 5,890.5 - 5,000 = \text{Rs. } 890.5$$

$$\text{C.I.} = \text{Rs. } 890.50$$

9.3 Computation of Principal



Example 8

Find sum of money that amounts to Rs. 5,000 after 2 years and to Rs. 10,000 after 4 years on compound interest.

Solution: - Given,

$$A_1 = \text{Rs. } 5,000; A_2 = \text{Rs. } 10,000; n_1 = 2 \text{ years; and } n_2 = 4 \text{ years}$$

$$P = ?$$

$$\text{We know that } A_1 = P \left(1 + \frac{R}{100}\right)^{n_1} \dots\dots\dots (1)$$

$$\text{And, } A_2 = P \left(1 + \frac{R}{100}\right)^{n_2} \dots\dots\dots (2)$$

From Eq. (1) and (2)

$$\text{Or, } \frac{A_1}{A_2} = \frac{\left(1 + \frac{R}{100}\right)^{n_1}}{\left(1 + \frac{R}{100}\right)^{n_2}} = \frac{\left(1 + \frac{R}{100}\right)^2}{\left(1 + \frac{R}{100}\right)^4} = \frac{1}{\left(1 + \frac{R}{100}\right)^2} = \frac{5000}{10000} = \frac{1}{2}$$

$$\text{Or, } \left(1 + \frac{R}{100}\right)^2 = 2 \dots\dots\dots (3)$$

From Eq. (1)

$$A_1 = 5000 = P \left(1 + \frac{R}{100}\right)^{n_1} = P \left(1 + \frac{R}{100}\right)^2$$

From Eq. (3)

$$P = \frac{5000}{2} = \text{Rs. } 2,500$$



Example 9

Find the sum of money which will amount to Rs. 26010 in 6 months at the rate of 8% p.a. when the interest is compounded half-yearly.

Solution: - Given,

$$A = \text{Rs. } 26,010; n = 6 \text{ months} = \frac{6}{12} \text{ Year} = \frac{1}{2} \text{ Year, } R = 8\%$$

When interest is compounded Half-yearly

$$\text{Then Rate of interest for half year } R_H = \frac{R}{2} = 4\%$$

$$\text{And time for half year } n_H = 2n = 1$$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n} = P \left(1 + \frac{4}{100}\right)^1 = 26010$$

$$\text{Or, } P = \frac{26010}{1.04} = \text{Rs. } 25009.6$$

9.4 Computation of Rate or Rate of Interest



Example 10

At what rate percent p.a. of compound interest will Rs.16,000 amount to Rs.1, 21,500 in 5 years?

Solution: - Given,

$P = \text{Rs.}16,000$; $A = \text{Rs.}1,21,500$; and $n = 5$ years

$R = ?$

We know that $A = P \left(1 + \frac{R}{100}\right)^n$

$$\text{Or, } \frac{A}{P} = \frac{121500}{16000} = \frac{243}{32} = \left(\frac{3}{2}\right)^5 = \left(1 + \frac{R}{100}\right)^5$$

$$\text{Or, } \frac{3}{2} - 1 = \frac{1}{2} = \frac{R}{100}$$

Or, $R = 50\%$ p.a.



Example 11

At what rate percent p.a. of compound interest will Rs.16,000 amount to Rs. 17,640 in 2 years?

Solution - Given,

$P = \text{Rs.}16,000$; $A = \text{Rs.}17,640$; $n = 2$ years

$R = ?$

We know that $A = P \left(1 + \frac{R}{100}\right)^n$

$$\text{Or, } \frac{A}{P} = \frac{17640}{16000} = \frac{1764}{1600} = \left(\frac{42}{40}\right)^2 = \left(1 + \frac{R}{100}\right)^2$$

$$\text{Or, } \frac{42}{40} - 1 = \frac{2}{40} = \frac{1}{20} = \frac{R}{100}$$

Or, $R = 5\%$ p.a.



Example 12

A sum of money Rs. 5,760 amounts to Rs. 6,250 in 1 year compounded half-yearly. Compute rate of interest p.a.

Solution - Given,

$P = \text{Rs.} 5,760$; $A = \text{Rs.} 6,250$; and $n = 1$ years

$R = ?$

We know that When interest is compounded Half-yearly

Then Rate of interest for half year $R_H = \frac{R}{2}$

Analytical Skills

And time for half year $n_H = 2n = 2 \times 1 = 2$

Therefore, Amount = $P \left(1 + \frac{R/2}{100}\right)^{2n} = P \left(1 + \frac{R}{200}\right)^2$

$$\text{Or, } \frac{A}{P} = \frac{6250}{5760} = \frac{625}{576} = \left(\frac{25}{24}\right)^2 = \left(1 + \frac{R}{200}\right)^2$$

$$\text{Or, } \frac{25}{24} - 1 = \frac{1}{24} = \frac{R}{200}$$

Or, $R = 8.333\%$ p.a.

**Example 13**

A sum of money Rs. 14,641 amounts to Rs. 20,736 in 1 year compounded quarterly. Compute rate of interest p.a.

Solution: - Given,

$P = \text{Rs. } 14,641$; $A = \text{Rs. } 20,736$; $n = 1$ years (4 quarters)

$R = ?$

When interest is compounded Quarterly

Then Rate of interest for Quarterly $R_Q = \frac{R}{4}$

And time for Quarter year $n_Q = 4n$

Therefore, Amount (A) = $P \left(1 + \frac{R/4}{100}\right)^{4n} = P \left(1 + \frac{R}{400}\right)^{4 \times 1}$

$$\text{Or, } \frac{A}{P} = \frac{20736}{14641} = \left(\frac{12}{11}\right)^4 = \left(1 + \frac{R}{400}\right)^4$$

$$\text{Or, } \frac{12}{11} - 1 = \frac{1}{11} = \frac{R}{400}$$

$$\text{Or, } R = \frac{400}{11} = 36\frac{4}{11}\% \text{ p.a.}$$

**Example 14**

At what rate per cent compound interest does a sum of money become nine-fold in 2 years

Solution - Given, $A = 9P$; $n = 2$ years

$R = ?$

We know that $A = P \left(1 + \frac{R}{100}\right)^n$

$$\text{Or, } \frac{A}{P} = \frac{9P}{P} = \left(\frac{3}{1}\right)^2 = \left(1 + \frac{R}{100}\right)^2$$

$$\text{Or, } 3 - 1 = 2 = \frac{R}{100}$$

Or, $R = 200\%$ p.a.

**Example 15**

A sum of money at compound interest amounts to Rs.800 in three year and to Rs.1,250 in five years.
Find the rate of interest p.a.

Solution - Given,

$$A_1 = \text{Rs. } 800; A_2 = \text{Rs. } 1250; n_1 = 3 \text{ years; and } n_2 = 5 \text{ years}$$

$$R = ?$$

$$\text{We know that } A_1 = P \left(1 + \frac{R}{100}\right)^{n_1} \dots\dots\dots (1)$$

$$\text{And, } A_2 = P \left(1 + \frac{R}{100}\right)^{n_2} \dots\dots\dots (2)$$

From Eq. (1) and (2)

$$\text{Or, } \frac{A_1}{A_2} = \frac{\left(1 + \frac{R}{100}\right)^{n_1}}{\left(1 + \frac{R}{100}\right)^{n_2}} = \frac{\left(1 + \frac{R}{100}\right)^3}{\left(1 + \frac{R}{100}\right)^5} = \frac{1}{\left(1 + \frac{R}{100}\right)^2} = \frac{800}{1250} = \frac{16}{25}$$

$$\text{Or, } \left(1 + \frac{R}{100}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2 \dots\dots\dots (3)$$

$$\text{Or, } \frac{5}{4} - 1 = \frac{1}{4} = \frac{R}{100}$$

$$\text{Or, } R = 25\% \text{ p.a.}$$

**Example 16**

A sum of money placed at C.I. doubles in 3 years. In how many years will it become four times?

Solution - Given, $A_1 = 2P$; $A_2 = 4P$; $n_1 = 3$ years; and $n_2 = ?$

We know that if a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Here $n = 2$, $t = 3$ years, and $m = 2$ (four times i.e., $2 \times 2 = 2^2$ or n^m)

$$T = mt = ?$$

$$T = m \times t$$

$$T = 2 \times 3$$

$$T = 6 \text{ years}$$

**Example 17**

A sum of money doubles itself at C.I. in 15 years. In how many years will it become eight times?

Solution - Given,

$n = 2$, $t = 15$ years, and $m = 3$ (Eight times i.e., $2 \times 2 \times 2 = 2^3$ or n^m)

$$T = ?$$

Analytical Skills

We know that if a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

$$T = m \times t$$

$$T = 3 \times 15$$

$$T = 45 \text{ years}$$

**Example 18**

In how many years Rs.1,00,000 will become Rs.1,33,100 at compound interest rate of 10% p.a.?

Solution – Given,

$P = \text{Rs. } 1,00,000$; $R = 10\% \text{ p.a.}$; and $A = \text{Rs. } 1,33,100$

$n = ?$

We know that $A = P \left(1 + \frac{R}{100}\right)^n$

$$\text{Or, } \frac{A}{P} = \frac{133100}{100000} = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3 = \left(1 + \frac{R}{100}\right)^n = \left(1 + \frac{10}{100}\right)^n = \left(\frac{11}{10}\right)^n$$

Or, $n = 3$ years

9.5 Computation of Amount**Example 19**

Mehak invested an amount of Rs. 18,000 at compound interest rate 4% p.a. for a period of 3 years. What amount will she receive at the end of 3 years?

Solution – Given,

$P = \text{Rs. } 18,000$; $R = 4\% \text{ p.a.}$; and $n = 3$ years

$A = ?$

$$\text{We know that } A = P \left(1 + \frac{R}{100}\right)^n = 18000 \left(1 + \frac{4}{100}\right)^3 = 18000 \times \left(\frac{26}{25}\right)^3 = 20247.552$$

Or, $A = \text{Rs. } 20,247.552$

**Example 20**

Rohit invested an amount of Rs. 1,000 at 4% p.a. compounded half-yearly for a period of 2 years. What amount will he receive at the end of 2 years?

Solution – Given,

$P = \text{Rs. } 1,000$; $R = 4\% \text{ p.a. or } (2\% \text{ half-yearly})$; and $n = 2$ years (4 half-years)

$A = ?$

$$\text{Therefore, Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n} = 1000 \left(1 + \frac{2}{100}\right)^4 = 1000(1.02)^4 = \text{Rs. } 1082.43$$

**Example 21**

Neha invested an amount of Rs. 24,000 at 12% p.a. compounded quarterly for a period of 1 year.

What amount will she receive at the end of 1 year?

Solution - Given,

$P = \text{Rs. } 24,000$; $R = 12\%$ p.a. or, (3 % per quarter); and $n = 1$ year (4 quarters)

$A = ?$

$$\text{We know that } P \left(1 + \frac{R/4}{100}\right)^{4n} = 24000 \left(1 + \frac{3}{100}\right)^4 = 24000(1.03)^4 = \text{Rs. } 27,012.21$$

Therefore, Amount = Rs. 27,012.21

**Example 22**

Find the amount on Rs. 4,000 at 10% p.a. for 2 years 6 months, compounded annually.

Solution - Given,

$P = \text{Rs. } 4,000$; $R = 10\%$ p.a.; and $n = 2$ years 6 months $= 2\frac{1}{2}$ years; $A = ?$

$$\begin{aligned} \text{Amount for fraction } 2\frac{1}{2} \text{ years} &= P \left(1 + \frac{R}{100}\right)^2 \times \left(1 + \frac{R/2}{100}\right) \\ &= 4000 \left(1 + \frac{10}{100}\right)^2 \times \left(1 + \frac{5}{100}\right) = 4000 \times 1.1^2 \times 1.05 = \text{Rs. } 5082 \end{aligned}$$

Therefore, Amount = Rs. 5082



Example 23 Amit invests Rs. 5,000 in a bond which gives interest at 2% p.a. during the first year, 5% p.a. during the second year and 10% p.a. during the third year. How much does he get at the end of the third year?

Solution - Given,

$P = \text{Rs. } 5,000$; $R_1 = 2\%$ p.a.; $R_2 = 5\%$ p.a.; and $R_3 = 10\%$ p.a.

$A = ?$

$$\begin{aligned} \text{We know that Amount for different yearly rate of interest} &= P \times \left(1 + \frac{R_1}{100}\right) \times \left(1 + \frac{R_2}{100}\right) \times \left(1 + \frac{R_3}{100}\right) \\ &= 5000 \times \left(1 + \frac{2}{100}\right) \times \left(1 + \frac{5}{100}\right) \times \left(1 + \frac{10}{100}\right) = 5000 \times (1.02) \times (1.05) \times (1.1) = \text{Rs. } 5890.50 \end{aligned}$$

$A = \text{Rs. } 5890.50$

**Example 24**

Analytical Skills

If a sum of Rs. 13040 is to be paid back in two equal annual instalments at $3\frac{3}{4}\%$ p.a., what is the amount of each instalment?

Solution – Given,

P = Rs. 13,040; R = $3\frac{3}{4}\%$ p.a. = $\frac{15}{4}\%$ p.a.; and No. of instalments = 2; Amount of each instalment = ?

We know that Amount of each instalment =

$$\frac{P}{\left(\frac{100}{100+R}\right)^1 + \left(\frac{100}{100+R}\right)^2} = \frac{13040}{\left(\frac{400}{400+15}\right)^1 + \left(\frac{400}{400+15}\right)^2} = \text{Rs. } 6889$$

9.6 Relation between Compound and Simple Interest



Example 25

What will be the difference between simple and compound interest on a sum of Rs. 30,000 put for 2 years at 5% p.a.?

Solution – Given,

P = Rs. 30,000; R = 5% p.a.; and n = 2 years

C.I. – S.I. = ?

We know that $(C.I. - S.I.)_{2 \text{ years}} = P \left(\frac{R}{100}\right)^2 = 30000 \times \left(\frac{5}{100}\right)^2 = 30000 \times \left(\frac{1}{20}\right)^2 = \frac{30000}{400} = \text{Rs. } 75$



Example 26

If the difference between compound interest & simple interest on a certain sum of money for 2 years at 5% p.a. is Rs. 100, find the sum.

Solution – Given,

n = 2; R = 5% p.a.; and $(C.I. - S.I.)_{2 \text{ years}} = \text{Rs. } 100$

P = ?

We know that $(C.I. - S.I.)_{2 \text{ years}} = P \left(\frac{R}{100}\right)^2 = P \times \left(\frac{5}{100}\right)^2 = \frac{P}{400} = 100$

Or, P = Rs. 40,000



Example 27

Find the rate percent if the difference between the compound interest and simple interest on Rs. 40,000 for 2 years is Rs. 100.

Solution – Given,

P = Rs. 40,000; n = 2; $(C.I. - S.I.)_{2 \text{ years}} = \text{Rs. } 100$

R = ?

We know that $(C.I. - S.I.)_{2 \text{ years}} = P \left(\frac{R}{100} \right)^2 = 40,000 \times \left(\frac{R}{100} \right)^2 = 4 \times R^2 = 100$

Or, R = 5% p.a.



Example 28

Find the rate percent if the S.I. on a certain sum of money for 2 years is Rs. 4,000 and difference between the compound interest and simple interest is Rs. 100.

Solution - Given,

n = 2; C.I. - S.I. = Rs. 100; and S.I. = Rs. 4,000

R = ?

We know that $S.I._{2 \text{ years}} = \frac{P \times R \times 2}{100} = \text{Rs. } 4,000$

Or, $\frac{P \times R}{100} = \text{Rs. } 2,000$

$(C.I. - S.I.)_{2 \text{ years}} = P \left(\frac{R}{100} \right)^2 = \frac{P \times R}{100} \times \frac{R}{100} = 100$

Or, $2000 \times \frac{R}{100} = 100$

Or, R = 5% p.a.



Example 29

What will be the difference between simple and compound interest on a sum of Rs. 8,000 put for 3 years at 5% p.a.?

Solution - Given,

P = 8,000; R = 5% p.a.; and n = 3 years

C.I. - S.I. = ?

We know that $(C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100} \right)^3 + 3P \left(\frac{R}{100} \right)^2$

Or $(C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100} \right)^2 \left(\frac{R}{100} + 3 \right) = 8000 \times \left(\frac{5}{100} \right)^2 \times \left(\frac{5}{100} + 3 \right)$
 $= 8000 \times 0.05^2 \times 3.05 = \text{Rs. } 61$



Example 30

The difference between the compound interest and simple interest on a certain sum of money for 3 years at 10% p.a. is Rs. 93. Find the sum.

Solution - Given,

n = 3; R = 10% p.a.; and C.I. - S.I. = Rs. 93

Analytical Skills

P = ?

We know that $(C.I. - S.I.)_{3 \text{ years}} = P \left(\frac{R}{100} \right)^2 \left(\frac{R}{100} + 3 \right) = P \times \left(\frac{10}{100} \right)^2 \times \left(\frac{10}{100} + 3 \right) = 93$

$$P = \frac{93 \times 100 \times 10}{31} = \text{Rs. } 3000$$

**Example 31**

The difference between compound and simple interests on a certain sum of money at the interest rate of 10% per annum for $1\frac{1}{2}$ years is Rs. 183, when the interest is compounded half yearly. Find the sum of money.

Solution - Given,

$n = 1\frac{1}{2} \text{ years} = \frac{3}{2} \text{ years} = 3 \text{ (half years)}$; $R = 10\% \text{ p.a.} = 5\% \text{ (half yearly)}$; and **C.I. - S.I. = Rs. 183**

P = ?

We know that $C.I. - S.I. = \left[P \left\{ \left(1 + \frac{5}{100} \right)^3 - 1 \right\} \right]_{C.I.} - \left(\frac{P \times 10 \times 3}{200} \right)_{S.I.} = 183$

$$\text{Or, } P \times 0.157625 - \frac{P \times 3}{20} = 183$$

Or, P = Rs. 24000

Summary

The key concepts learned from this unit are: -

We understood the basic concept of C.I.

We solved formula-based questions of C.I.

We understood the difference between S.I. and C.I. and solved problems based on them.

Keywords

- Principal
- Rate
- Time
- Compound Interest
- Amount
- Difference between C.I. and S.I.

Self Assessment

1. What would be the compound interest accrued on an amount of Rs. 8000 at the rate of 15%. p.a. in 3 years?

- A. Rs. 4051
- B. Rs. 4167
- C. Rs. 4283
- D. Rs. 4325

2. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in
- A. 10 years
 - B. 20 years
 - C. 12 years
 - D. 15 years
3. The compound interest on Rs. 2800 for 18 months at 10% p.a. is
- A. Rs. 420
 - B. Rs. 434
 - C. Rs. 436.75
 - D. Rs. 441.35
4. Find the present worth of Rs. 9261 due 3 years, hence at 5% per annum compounded yearly.
- A. Rs.7000
 - B. Rs.8000
 - C. Rs.9000
 - D. Rs.1000
5. A man saves ` 200 at the end of each year and lends the money at 5% compound interest. How much will it become at the end of 3 years?
- A. Rs. 565.25
 - B. Rs. 635
 - C. Rs. 662.02
 - D. Rs. 666.50
6. A sum put out at 4% compound interest payable half-yearly amounts to `6632.55 in 1 years. The sum is
- A. Rs. 6530
 - B. Rs. 6250
 - C. Rs. 6470
 - D. Rs. 6523
7. What will be the compound interest accrued on an amount of Rs.10000 @ 20 p.c. p.a. in 2 years if the interest is compounded half-yearly?
- A. Rs. 4400
 - B. Rs. 4600
 - C. Rs. 4641
 - D. Rs. 4680
8. The difference of compound interest on `800 for 1 year at 20% per annum when compounded half-yearly and quarterly is
- A. Rs. 4.40
 - B. Rs. 3.40
 - C. Rs. 6.40
 - D. Rs. 5.40
9. Find the compound interest on Rs. 15625 for 9 months at 16% per annum compounded quarterly.
- A. Rs. 1851
 - B. Rs. 1941
 - C. Rs. 1951
 - D. Rs. 1961
10. Rs. 800 at 5% per annum compound interest amount to Rs. 882 in how many years?

Analytical Skills

- A. 6 years
- B. 2 years
- C. 4 years
- D. 5 years

11. The simple interest accrued on an amount of Rs. 20000 at the end of 3 years is Rs. 7200. What would be the compound interest accrued on the same amount at the same rate in the same period?

- A. Rs. 8098.56
- B. Rs. 8112.86
- C. Rs. 8246.16
- D. Rs. 8342.36

12. What will be the amount if a sum of Rs.5000 is placed at compound interest for 3 years while rate of interest for the first, second and third years is 2, 3 and 4 per cent, respectively?

- A. Rs. 5643.12
- B. Rs. 5463.12
- C. Rs. 6413.12
- D. Rs. 6553.22

13. A certain sum will amount to Rs.12,100 in 2 years at 10% per annum of compound interest, interest being compounded annually. The sum is

- A. Rs. 12000
- B. Rs. 6000
- C. Rs. 8000
- D. Rs. 10000

14. The compound interest on Rs. 2000 at 5% per annum, compounded yearly, for 2 years is

- A. Rs. 315
- B. Rs. 425
- C. Rs. 205
- D. Rs. 334

15. A man borrows money at 3% per annum interest payable yearly and lend it immediately at 5% interest (compound) payable half-yearly and thereby gains Rs. 330 at the end of the year. The sum borrowed is

- A. Rs. 17,000
- B. Rs.16,500
- C. Rs.15,000
- D. Rs.16,000

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. D | 3. B | 4. B | 5. C |
| 6. B | 7. C | 8. A | 9. C | 10. B |
| 11. A | 12. B | 13. D | 14. C | 15. D |

Review Questions

1. Find compound interest on Rs. 5000 for 2 years at 4% p.a.
2. At what rate of compound interest per annum will a sum of Rs. 1200 become Rs. 1348.32 in 2 years?

3. Divide Rs. 8840 between A and B so that the amount received by A at the end of 8 years may be equal to the amount received by B at the end of 10 years, compound interest being at 10% per annum.
4. Find the compound interest on Rs. 1000 at 40% per annum compounded quarterly for 1 year.
5. Find the compound interest on Rs. 4000 at 24% per annum for 3 months, compounded monthly.
6. The compound interest on a sum of money for 3 years at 5% is Rs. 1324.05. What is the simple interest?
7. A sum of money placed at compound interest doubles in 3 years. In how many years will it become four times?
8. If the difference of the compound interest on a sum of money for 3 years is Rs. 186. Find the sum of money if the rate of interest in both cases be 10%.
9. A sum of money at compound interest amounts to Rs. 4050 in one year and to Rs. 4723.92 in 3 years. Find the rate of interest per annum.
10. If the difference of the compound interest on a sum of money for 3 years is Rs. 186. Find the sum of money if the rate of interest in both cases be 10%.



Further Reading

1. Quantitative Aptitude for Competitive Examinations by Dr. R. S. Aggarwal, S. Chand Publishing.
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3. Magical Book on Quicker Maths by M Tyra, Banking Service Chronicle.
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Web Links

1. <https://www.hitbullseye.com/quant>
2. <https://www.indiabix.com/aptitude/questions-and-answers/>
3. <https://www.examveda.com/mcq-question-on-arithmetic-ability/>

Unit 10: Alphabet Test

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Alphabetical order of words and letter word problems.
- Solve problems based on alphabetical order of words and letter word problems.
- Word formation by unscrambling letters.
- solve problems based on alphabet test of word formation by unscrambling letters and alphanumeric puzzles.

Introduction

Arranging words in alphabetical order implies 'to arrange them in the order as they appear in a dictionary', i.e., as per the order in which the beginning letters of these words appear in the English alphabet.

10.1 Alphabet Test

Alphabet Test is one of the easiest and important concepts of General Mental Ability Segment of Reasoning. In this type of question, students are asked to find the place of an alphabet or a word based on the different types of arrangement. Here, you will learn different forms of Alphabet Arrangements like arranging Words in Alphabetical Order, problems based on Letter-Word, Alphabetical Quibble and New Word Formation, etc.

10.2 Type of Alphabet Arrangement

TYPE 1: Arranging Words in ALPHABETICAL ORDER

Arranging words in alphabetical order implies arranging words as per the order in which the beginning letters of those words appear in the English Alphabet or to arrange them in the order as they appear in a dictionary.



Example: Consider the words – Theoretical, Convenient, Frequent, Deficit, Arrange

These words begin with letters T, C, F, D, A

Their order in English alphabet is A, C, D, F, T

Thus, the correct alphabetical order of these words is: Arrange, Convenient, Deficit, Frequent, Theoretical.

In Some cases, two or more words begin with the same letter. Such words should be arranged in the order of second letters in the alphabet.



Example: Which of the following words will come second in the English dictionary?

- a) Magical
- b) Magnify
- c) Maternal
- d) Marshal
- e) Magnetic

Explanation: The given words can be arranged in the alphabetical order as:

Magical, Magnetic, Magnify, Marshal, Maternal.

Clearly, 'Magnetic' comes second. So, the answer is 'e'.



Example: In the following question five words are given. Find out which word will come in the middle, if all the five words are arranged alphabetically as in a dictionary.

- a) Savour
- b) Save
- c) Savage
- d) Sausage
- e) Saviour

Answer: b)

Explanation: If we arrange the words in alphabetical order, then the word 'Save' will come in the middle:

Sausage, Savage, Save, Saviour, Savour

TYPE 2: Problems based on LETTER-WORD

Example: How many such letters are there in the word “ACCELERATION”, each of which is as far away from the beginning of the word as it is from the beginning of the English Alphabet?

- a) None
- b) One
- c) Two
- d) Three

Answer: b)

Explanation: Clearly, C is the third letter in the word “ACCELERATION” as well as in the English Alphabet. Therefore, there is only one such letter.



Example: How many such pairs of letters are there in the word CORPORATE each of which has as many letters in the same sequence between them in the word as in the English alphabet?

- a. None
- b. 1
- c. 2
- d. 3
- e. more than 3

Explanation:

C O R P O R A T E

Three pairs- (P,R), (R,T) and (P,O) have as many letters between them in the word as in the English alphabet. But since the letters must be in the same sequence in the word as in the English alphabet, so the desired pairs are (P,R) and (R,T) only.



Example: If the last four letters of the word CONCENTRATION are written in reverse order followed by next two in the reverse order and next three in the reverse order and then followed by first four in the reverse order, counting from the left, which letter would be eighth in the new arrangement?

- a) N
- b) T
- c) E
- d) R

Explanation: The new letter sequence is NOITARTNECNOC.

The eighth letter from the left is N

TYPE 3: Problems based on ALPHABETICAL QUIBBLE**Example:** Answer this question based on the following English Alphabet:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

If in the English alphabet every fourth letter is replaced by the symbol (#), which of the following would be ninth to the left of the fourteenth element from the left?

- a) E
- b) #
- c) W
- d) F

Answer: a)

Explanation: If every fourth alphabet is replaced by # then new series becomes:

A B C # E F G # I J K # M N O # Q R S # U V W # Y Z

Here, the fourteenth element from the left is N. The ninth element to the left of N is E.

TYPE 4: NEW WORD FORMATION using letters of a word given in the problem

Example: If it is possible to make meaningful words with the second, the third, the sixth and the eighth letter of the word 'FRAGMENT', using each letter only once, then how many new words can be formed with these letters?

- a) 1
- b) 2
- c) 3
- d) 4

Answer: c)

Explanation: The second, the third, the sixth and the eighth letter of the word FRAGMENT are R, A, E and T, respectively. The three new words formed will be TEAR, TARE and RATE.

TYPE 5: WORD FORMATION BY UNSCRAMBLING LETTERS

In this type of questions, a set of English letters is given in a jumble order. The student is required to arrange these letters to form a meaningful word.



Example: Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

T I R B H G

- a) 1 2 3 4 5 6

- b) 1, 3, 2, 4, 6, 5.
 c) 3, 2, 6, 5, 3, 1.
 d) 4, 3, 2, 6, 5, 1.
 e) 4, 5, 2, 3, 6, 1.

Ans: C 3, 2, 6, 5, 3, 1.

BRIGHT

TYPE 6: WORD FORMATION USING LETTERS OF A GIVEN WORD:



Example: If the letters of the word ARTICULATES can be used as many times as one wants to use, then which one of the following four words cannot be formed?

- a) COURTS
 b) LATER
 c) ELECTRIC
 d) ARTICLE

Explanation: Clearly, the word ARTICULATES does not contain the letter O and as such, the word COURTS cannot be formed. Hence, the answer is 'a'.

TYPE 6: ALPHA-NUMERIC SEQUENCE PUZZLE:



Example: R E 5 D A P \$ 3 T I Q 7 9 B # 2 K % U 1 M W 4 * J 8 N

How many such numbers are there in the above arrangement, each of which is immediately preceded by a consonant and not immediately followed by a consonant?

- a) None
 b) One
 c) Two
 d) Three
 e) None of these

Ans. C

Explanation: Clearly, the numbers 7 and 4 starting with consonant and not followed by consonants.
 Q 7 9; W 4 *.

10.3 Logical Sequence of Words

'Logical Order of Words' is basically the arrangement of words according to a certain order which can be their size, occurrence, dictionary order etc. In these types of questions, four/five/six words are given and the candidate is required to arrange the given words either in a logical sequence or in an order according to dictionary.

10.4 Type of Logical Sequence Questions

Type 1: Arrange According to Logic

As the name implies, in this type of questions, a sequence is formed with a certain number of words given in such a way that it gives a logical step-by-step completion of the process or the activity described.



Example: Arrange the given words in a logical and meaningful order.

1. Frog 2. Eagle 3. Grasshopper 4. Snake 5. Grass

(a) 3, 4, 2, 5, 1 (b) 1, 3, 5, 2, 4

(c) 5, 3, 1, 4, 2 (d) 5, 3, 4, 2, 1

Solution: (c) From the given words, it is deduced that grass is eaten by grasshopper, grasshopper is eaten by frog, frog is eaten by snake and finally, eagle eats snake. So, the correct logical arrangement of words is 5, 3, 1, 4, 2.

Type 2: Arrangement According to Dictionary

Arranging words in alphabetical order implies 'to arrange them in the order as they appear in a dictionary'. For this arrangement, first we shall take the first letter of each word and then arrange the words in the order in which they appear in the English alphabet, then take the second letter and so on.



Example: Arrange the following words according to dictionary.

1. Fenestration 2. Feather 3. Feed head 4. Feature 5. Feminine

(a) 4, 2, 3, 5, 1

(b) 2, 4, 1, 5, 3

(c) 2, 4, 3, 5, 1

(d) 4, 2, 3, 1, 5

Solution: (c) Sequence of given words as per dictionary is as follows

Feather ---> Feature ----> Feed head -----> Feminine ----> Fenestration, i.e., 2, 4, 3, 5, 1

Summary

The key concepts learnt from this Unit are: -

We have learnt about key concepts of Alphabet test and logical sequence.

We have learnt tricks to solve different types of Alphabet test and logical sequence.

We have learnt to solve the questions with meaning and according to dictionary.

Keywords

- Alphabet test
- Logical Sequence

- Word Formation
- Unscrambling Letters

Self Assessment

Directions: Which one of the given responses would be a meaningful order of the following words?

1. (a) Honey(b) Flower(c) Bee(d) Wax.

- A. b, a, d, c
- B. b, c, a, d
- C. d, c, b, a
- D. a, c, d, b

2. (a) Plant(b) Food(c) Seed(d) Leaf(e) Flower

- A. a, c, d, e, b
- B. c, b, d, e, a
- C. c, a, d, e, b
- D. e, d, c, b, a

3. a. Probation, b. Interview c. Selection, d. Appointment, e. Advertisement, f. Application

- A. e, f, c, b, d, a
- B. e, f, d, b, c, a
- C. f, e, d, b, c, a
- D. e, f, b, c, d, a

4. a. Doctor, b. Fever, c. Prescribe, d. Diagnose, e. Medicine

- A. b, a, c, d, e
- B. a, d, c, b, e
- C. b, a, d, c, e
- D. b, d, c, e, a

5. Which one of the given responses would be a meaningful order of the following?

1. Orange, 2. Indigo, 3. Red, 4. Blue, 5. Green, 6. Yellow, 7. Violet

- A. 7, 2, 4, 5, 6, 1, 3
- B. 7, 2, 4, 6, 5, 1, 3
- C. 7, 2, 6, 4, 5, 1, 3
- D. 7, 2, 6, 4, 1, 5, 3

6. Arrange the following words in a meaningful order:

- 1. Brother
- 2. Husband
- 3. Father
- 4. Son

Analytical Skills

5. Son-in-law

A. 3, 2, 1, 5, 4

B. 4, 1, 2, 5, 3

C. 4, 1, 5, 2, 3

D. 3, 1, 4, 2, 5

7. Which one of the following words will come fourth in the Dictionary?

Propriety, Proposition, Prosecute, Proposal, Prosody.

A. Proposition

B. Prosody

C. Proposal

D. Prosecute

8. Arrange the following words according to English Dictionary.

(1) PREMONITION

(2) PRELUDE

(3) PREMICE

(4) PRELIMINARY

(5) PREMIUM

A. 42153

B. 24351

C. 42351

D. 24135

9. Arrange the following words according to dictionary:

a. Brush, b. Bread, c. Broad, d. Border, e. Butter

A. d, a, b, c, e

B. d, b, c, a, e

C. d, b, a, c, e

D. d, c, b, a, e

10. Arrange the following words as per order in the dictionary:

a. Detach

b. Devise

c. Denote

d. Digest

e. Depict

A. e, d, c, b, a

B. c, e, a, b, d

C. c, b, a, e, d

D. e, b, d, a, c

11. Arrange the following words as per order in the dictionary.

1. Billian
 2. Bifurcate
 3. Bilateral
 4. Bilirubin
- A. 2, 1, 3, 4
B. 4, 3, 2, 1
C. 2, 3, 4, 1
D. 2, 3, 1, 4

12. Arrange the following according to dictionary.

1. Fenestration
 2. Feather
 3. Feed head
 4. Feature
 5. Feminine
- A. 4, 2, 3, 5, 1
B. 4, 2, 3, 1, 5
C. 2, 4, 3, 5, 1
D. 2, 4, 1, 5, 3

13. Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

E H R A S P

1 2 3 4 5 6

- A. 2, 4, 6, 1, 3, 5
B. 3, 4, 2, 1, 6, 5
C. 5, 2, 4, 6, 1, 3
D. 6, 2, 3, 4, 5, 1

14. Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

T L P N A E

1 2 3 4 5 6

- A. 3, 2, 5, 4, 1, 6
B. 3, 2, 5, 4, 6, 1
C. 4, 5, 3, 6, 2, 1
D. 4, 6, 1, 3, 5, 2

15. R E 5 D A P \$ 3 T I Q 7 9 B # 2 K % U 1 M W 4 * J 8 N

Which of the following is exactly in the middle between 3 and 1 in the above arrangement?

- A. B

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- B. K
- C. 9
- D. #

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. D | 4. C | 5. A |
| 6. A | 7. D | 8. C | 9. B | 10. B |
| 11. C | 12. C | 13. D | 14. B | 15. A |

Review Questions

1. Arrange the following words as per their order in the dictionary:

- 1. PHYSICAL
- 2. PHYSICS
- 3. PHYSIOLOGY
- 4. PHYSICIAN
- 5. PHONE

(1) 5, 1, 4, 2, 3

(2) 5, 1, 2, 3, 4

(3) 1, 4, 5, 2, 3

(4) 1, 5, 4, 3, 2

2. Arrange the following words as per order in the dictionary:

- 1. Ambitions
- 2. Ambiguous
- 3. Ambiguity
- 4. Animation
- 5. Animal

(1) 3, 2, 4, 1, 5

(2) 3, 2, 5, 4, 1

(3) 3, 2, 1, 5, 4

(4) 3, 2, 4, 5, 1

3. Arrange the following words as per order in the dictionary:

- 1. Brittle
- 2. Brisk

3. Bright

4. Bride

(1) 4, 3, 2, 1

(2) 1, 2, 3, 4

(3) 2, 3, 4, 1

(4) 4, 2, 1, 3

4. Arrange the following words as per order in the dictionary:

1. Preach

2. Praise

3. Precinct

4. Precept

5. Precede

(1) 2, 1, 5, 4, 3

(2) 2, 1, 3, 4, 5

(3) 2, 5, 1, 4, 3

(4) 1, 2, 5, 4, 3

5. Arrange the following words as per order in the dictionary:

1. Follicle

2. Folk

3. Follow

4. Foliage

(1) 4, 2, 1, 3

(2) 3, 4, 2, 1

(3) 4, 3, 1, 2

(4) 2, 4, 3, 1

6. Arrange the following words as per order in the dictionary

1. Maternity

2. Matriarchy

3. Matchbox

4. Matricide

(1) 3, 1, 2, 4

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(2) 4, 3, 1, 2

(3) 3, 4, 1, 2

(4) 1, 3, 4, 2

7. Arrange the following words as per order in the dictionary

1. Launderette

2. Laughter

3. Laundry

4. Launch

(1) 4, 1, 2, 3

(2) 1, 3, 2, 4

(3) 4, 2, 1, 3

(4) 2, 4, 1, 3

8. Arrange the following words as per order in the dictionary.

1. Forecast

2. Forget

3. Foreign

4. Forsook

5. Force

(1) 3, 5, 1, 2, 4 (2) 5, 1, 3, 2, 4

(3) 5, 1, 3, 4, 2 (4) 5, 1, 2, 3, 4

9. Arrange the following words as per order in the dictionary.

1. Continuation

2. Contention

3. Contain

4. Continuous

5. Count

(1) 32415

(2) 32451

(3) 31245

(4) 32145

10. Which one of the given responses would be a meaningful order of the following?

1. Stone

2. Sand

3. Rock

4. Boulder

5. Hill

(1) 2, 1, 3, 4, 5 (2) 5, 3, 2, 1, 4

(3) 5, 4, 2, 1, 3 (4) 1, 4, 2, 3, 5

11. Arrange the words given below in a meaningful sequence:

1. Printer

2. Publisher

3. Writer

4. Editor

5. Seller

(1) 3, 4, 2, 1, 5 (2) 3, 4, 1, 2, 5

(3) 2, 4, 3, 5, 1 (4) 2, 3, 4, 1, 5

12. Arrange the words given below in a meaningful sequence:

1. Crop 2. Root 3. Stem

4. Seed 5. Flower

(1) 2 3 5 1 4

(2) 2 4 5 1 3

(3) 2 3 4 1 5

(4) 4 2 3 5 1

13. Select the combination of numbers so that the letters arranged accordingly in the form of meaningful word.

N A E H L D

1 2 3 4 5 6

a. 2, 1, 6, 4, 3, 5

b. 2, 6, 4, 3, 5, 1

c. 4, 2, 1, 6, 5, 3

d. 4, 3, 6, 5, 2, 1

14. How many meaningful words can be formed using the first, the third, the fifth and the sixth letters of the word TRADEMARK using each letter only once in each word? a. One

Analytical Skills

- b. Two
- c. Three
- d. Four e. More than four

15. £= β F 2 * K S 7 5 # \$ P L V 8 @ M U E 6 ∞ Q G © 9 3 & T Y ¥

How many such letters are there in the arrangement each of which is either immediately preceded by a symbol or immediately followed by a number, but not both?

- a. Three
- b. Four
- c. Five
- d. Six
- e. None of these

**Further Reading**

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

**Web Links**

1. <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
2. <https://www.hitbullseye.com/Reasoning>

Unit 11: Number Test

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- What is number test.
- Position switching of numbers.
- Problems based on number test.

Introduction

Arranging numbers in the specific given conditions.

11.1 Number Test

In this type of questions, generally a set, group or series of numerals is given and the candidate is asked to trace out numerals following certain given conditions. In these sorts of problems, a number, an arrangement of numbers, arrangement of digit is given and the applicant is requested that follow out digit taking after certain given conditions or lying at particular said positions in the wake of rearranging as indicated by a specific given example.



Example: How many such 5s are there in the following number sequence each of which is

immediately preceded by 3 or 4 but not immediately followed by 8 or 9?

3 5 9 5 4 5 5 3 5 8 4 5 6 7 3 5 7 5 5 4 5 2 3 5 1 0

(A) None

(B) Three

(C) Four

(D) Five

(E) None of these

Solution:

As we know, a number that comes after a given number is said to follow it while the one which comes before the given number precedes it.

Thus, the number satisfying the given conditions may be marked as follows:

35954 [5] 53584 [5] 673 [5] 7554 [5] 23 [5] 1 0

Clearly, there are five such 5's. Hence, the answer is (d).



Example: How many 5's is there in the following number sequence which are immediately preceded by 7 and immediately followed by 6?

Terms: 7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 6 4 3 2 5 6 7 8

(a) 1

(b) 2

(c) 3

(d) 4

Solution. (a) Here, 7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 6 4 3 2 5 6 7 8

Preceded by 7 and followed by 6 So, there is only one such 5.

756.

11.2 Position Switching of Numbers



Example: What will be last digit of the 3rd number from top when the numbers given below are arranged in descending order after reversing the position of the digits within each number?

517 325 639 841 792

(a) 2

(b) 5

(c) 7

(d) 3

Sol. (d) The given numbers are:

517 325 639 841 792

After reversing, the numbers become as follows: 715 523 936 148 297

When arranged in descending order the numbers become as follows:

936 715 523 297 148;

Now, the third number from top is 523. Hence, the last digit of 523 is 3.

∴ Option (d) is correct.



Example: Which of the following will be the second digit of the third number from the top when they are arranged in descending order, after the first digit in each number is changed to its next higher digit?

738 429 156 273 894

A) 2

B) 3

C) 5

D) 7

E) 9

Answer: Option A

Explanation:

The new numbers formed are: 838, 529, 256, 373, 994.

These, in descending order, are: 994, 838, 529, 373, 256.

The third number from the top is 529, and its second digit is 2.

11.3 Problems on Number Test



Example 1: The digits of each of the following five numbers are written in reverse order and five new number are obtained

513 726 492 865 149

Which of the following will be the middle digit of the third number from the top when the new numbers are arranged in descending order?

A) 1

B) 2

C) 6

D) 9

E) None of these

Answer: Option C

Explanation:

The new numbers, arranged in descending order are 941 627 568 315 294.

The third number from the top is 568 and its middle digit is 6.



Example 2: In the following sequence of instructions, 1 stands for Run, 2 stands for Stop, 3 stands for Go, 4 stands for Sit and 5 stands for Wait. If the sequence were continued, which instruction will come next

4 4 5 4 5 3 4 5 3 1 4 5 3 1 2 4 5 4 5 3 4 5 3

- A) Wait
- B) Sit
- C) Go
- D) Stop
- E) Run

Answer: Option E

Explanation:

The given sequence may be analysed as under:

4 / 45 / 453 / 4531 / 45312 / 45 / 453 / 4531

Following the above sequence, the next number is 1 which stands for 'Run'



Example 3: In the series given below, how many eights are there each of which is exactly divisible by its immediately preceding as well as succeeding numbers?

2 8 3 8 2 4 8 2 4 8 6 8 2 8 2 4 8 3 8 2 8 6

- A) One
- B) Two
- C) Three
- D) Four

Answer: Option B

Explanation:

There are two 8's which is exactly divisible by its immediately preceding as well as succeeding numbers. As given below:

28382 [482] 4868 [282] 4838286.



Example 4: How many such 6's is there in the following number sequence each of which is immediately preceded by 3 or 4 but not immediately followed by 8 or 9?

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

Solution:

Given number sequence is

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

A number which comes after a given number is said to follow it while the one which precedes the given number goes before it.

Thus, the numbers satisfying the given conditions may be marked (with red colour) as follows:

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

Therefore, there are five such 6"s.



Example 5: How many pairs of successive numbers have a difference of 3 each? If the series:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 4 2 1 4 2 6 3.

Solution:

Given series is:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 2 1 4 2 6 3.

Here,

$$7 - 4 = 3$$

$$4 - 1 = 3$$

$$8 - 5 = 3$$

$$5 - 2 = 3$$

$$4 - 1 = 3$$

$$6 - 3 = 3$$

So, the pairs of successive numbers having a difference of 3 can be shown below:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 2 1 4 2 6 3

Therefore, there are six such pairs

Summary

The key concepts learned from this unit are: -

- Understood the Number test and position switching type questions.
- We have learnt how to solve different types of questions on Number Test and position switching of numbers.

Keywords

- Number test
- Position switching of numbers

Self Assessment

1. How many 3's is there in the following sequence which is neither preceded by nor immediately followed by 9?

9 3 6 6 3 9 5 9 3 7 8 9 1 6 3 9 6 3 9

- A. One
- B. Two
- C. Three
- D. None of these

2. Count each 7 which is not immediately preceded by 5 but is immediately followed by either 2 or 3. How many such 7's is there?

5 7 2 6 5 7 3 8 3 7 3 2 5 7 2 7 3 4 8 2 6 7 8

- A. 2
- B. 3
- C. 4
- D. 5

3. How many 7's is there in the following series which are preceded by 6 which is not preceded by 8?

8 7 6 7 8 6 7 5 6 7 9 7 6 1 6 7 7 6 8 8 6 9 7 6 8 7

- A. Nil
- B. One
- C. Two
- D. None of these

4. In the series given below, count the number of 9's, each of which is not immediately preceded by 5 but is immediately followed by either 2 or 3. How many such 9's are there?

1 9 3 2 1 7 4 2 6 9 7 4 6 1 3 2 8 7 4 1 3 8 3 2 5 6 7 4 3 9 5 8 2 0 1 8 7 4 6 3

- A. one
- B. three
- C. five
- D. six

5. If it is possible to form a number with the second, the 5th and the 8th digits of the number 3 1 5 4 9 7 8 6, which is perfect square of a two digit even number, which of the following will be the 2nd digit of that even number?

- A. 1
- B. 4
- C. 6
- D. No such number can be formed

6. Which is the third number to the left of the number which is exactly in the middle of the following sequence of the numbers?

1 2 3 4 5 6 7 8 9 2 4 6 8 9 7 5 3 1 9 8 7 6 5 4 3 2 1

- A. 3
- B. 4
- C. 5
- D. 6

7. In the following series of numbers, find out how many times, 1, 3, and 7 have appeared together, 7 being in the middle and 1 and 3 on either side of 7?

2 9 7 3 1 7 3 7 7 1 3 3 1 7 3 8 5 7 1 3 7 7 1 7 3 9 0 6

- A. 3
- B. 4
- C. 5
- D. more than 5
- E. None of these.

8. Study the number series given below and answer the questions that follow:

7 8 9 7 6 5 3 4 2 8 9 7 2 4 5 9 2 9 7 6 4 7.

Which option have equal frequency?

- A. 2,5,3
- B. 2,4,5
- C. 3,7,5
- D. 8,6,5

9. In the series?

6 4 1 2 2 8 7 4 2 1 5 3 8 6 2 1 7 1 4 1 3 2 8 6

how many pairs of alternate numbers have a difference of 2?

- A. One
- B. Two
- C. Three
- D. Five

10. Thirty-six vehicles are parked in a parking lot in a single row. After the first car, there is one scooter. After the second car, there are two scooters. After the third car, there are three scooters and so on. Work out the number of scooters in the second half of the row.

- A. 10
- B. 12
- C. 15
- D. 17

Answers for Self Assessment

1. D 2. A 3. D 4. B 5. B
 6. B 7. A 8. D 9. D 10. C

Review Questions

1. How many 4's are there preceded by 7 but not followed by 3? 5 9 3 2 1 7 4 2 6 9 7 4 6 1 3 2 8

7 4 1 3 8 3 2 5 6 7 4 3 9 5 8 2 0 1 8 7 4 6 3.

2. Study the number series given below and answer the questions that follow:

7 8 9 7 6 5 3 4 2 8 9 7 2 4 5 9 2 9 7 6 4 7.

How many 7s are preceded by 9 and followed by 6?

3. If the first and second digits in the sequence 5981327438 are interchanged, also the third and fourth digits, the fifth and sixth digits and so on, which digit would be the seventh counting to your left?

4. The position of the second and the 8th digit of the number 39128564 are interchanged. The position of the first and sixth digits is interchanged and the positions of the third and seventh digits are interchanged. Which of the following will be third digit left of the 3 after the re-arrangement.

5. What will be the difference between the sum of the odd digits and the sum of the even digits in the number 857423?

6. How many such digits are there in the number 831729564, each of which is as far away from the beginning of the number as when the digits are arranged in descending order?

Directions of (Q. 7- 9) The following questions are based on three-digit numbers given below.:

986 467 347 836 757 564

7. What will be the result if the first digit of the highest number is divided by the first digit of the lowest number?

8. If '2' is subtracted to each of the odd digits of all numbers, which of the following number thus obtained is the highest number after this operation?

9. If all digits in each number are arranged in descending order within the number, how many numbers are the same as the actual number in the new arrangement?

10. If in '4143656795' we add two to all the even numbers and subtract one from all the odd numbers then, what is the sum of the numbers which is fourth from the left end and fourth from the right end?

**Further Reading**

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

**Web Links**

1. <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
2. <https://www.hitbullseye.com/Reasoning>

Unit 12: Ranking and Time Sequence Test

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Different types of Ranking Test.
- Different types of Time Sequence Test.
- Problems based on Ranking and Time Sequence Test.

Introduction

Ranking and Time Sequence section has all the different types of questions on Ranking Test, and Time Sequence Test.

We all know the basics of Time Sequence related problems.

1 Minute = 60 seconds

1 Hour = 60 minutes

1 Day = 24 hours

1 Week = 7 days

1 Month = 4 weeks

1 Year = 12 months

1 Ordinary year = 365 days

1 Leap year = 366 days

1 Century = 100 years.

Month and Total number of days

January 31

February 28

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March 31

April 30

May 31

June 30

July 31

August 31

September 30

October 31

November 30

December 31.

Other facts to be remembered

A day is the period of the earth's revolution on its axis.

A „Lunar month“ is the time taken the moon to travel round the earth. It is equal to nearly 28 days.

The time taken by the Earth to make one complete revolution is said to be a complete Year, which is equivalent to exactly 365.2425 days or 365 days, 5 hours, 48 minutes and 47- seconds nearly.

But ignoring the .2425 we generalize an ordinary year to be of 365 days.

What is Leap Year and How it is decided?

Leap year is decided based on extra 0.2425 days, which is assumed as 0.25 and after every fourth year it becomes complete one day. Since for finding leap year 0.2425 days is approximated as 0.25 days, therefore difference between actual and approximated values of day ($0.25 - 0.2425 = 0.0075 = \frac{3}{400}$) is compensated in every 100 years and out of 400 years 3 100th years (100, 200, 300) is non-Leap years. Therefore, after every fourth-year leap year is coming except century years. But in century years multiple 400 year is taken as leap year and all other century year is non leap year.

If the number of a given year is divisible by 4, it is a leap year. Hence, the years like 1996,2008,2012 are leap years. But years like 1997,1991, 2005,2007 are not divisible by 4 and therefore, such years are not leap years.

In a leap year, February has 29 days.

A leap year has 52 weeks and 2 days.

12.1 Ranking Test

The position of a thing/person etc. in a definite order is called as 'Rank'.

In this type of questions, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. However, sometimes this question is put in the form of a puzzle of interchanging seats by two persons. In these sorts of questions, generally the rank of a person both from the top and from the base are said and the aggregate number of persons is asked. In any case, in some cases this grouping is placed as a riddle of interchanging seats by two persons. In these types of questions, generally the ranks of a person both from the top and from the bottom is mentioned and the total number of persons is asked.

Type - 1: Rank of a person in a queue

Position of person from upward = [Total no. of persons - position of person from down] + 1.

Position of person from downward = [Total no. of persons - position of person from up] + 1.

Position of person from right = [Total no. of persons - position of person from left] + 1.

Position of person from left = [Total no. of persons - position of person from right] + 1.



Example: - Anita ranks twelfth in a class of forty-six. What will be her rank from the last?

Solution:

Rank of Anita from the last

$$= [\text{Total students} - \text{her rank from first}] + 1$$

$$= (46 - 12) + 1 = 35^{\text{th}}$$

Type - 2: Total no. of persons

Total no. of persons = [Position of person from upward/right + Position of person from downward/left] - 1.

$$\text{Total} = L + R - 1$$

$$\text{Total} = T + B - 1$$



Example: - Rakesh ranks 7th from the top and 28th from the bottom in a class. How many students are there in the class?

Solution:

$$\text{Total no. of students} = [7 + 28] - 1 = 34.$$



Example: - In a row of persons, position of A from left side of the row is 27th and position of A

from right side of the row is 34th. Find total no. of persons in the row?

Solution:

$$\text{Total no. of students} = (\text{Position of A from left} + \text{Position of A from right}) - 1$$

$$\Rightarrow \text{Total no. of students} = (27 + 34) - 1 = 61 - 1 = 60.$$

Type - 3: When two persons change their rank in a queue

If two persons are on a definite position from up and down (or left and right) and they interchange their ranks, then

$$\text{Total no. of persons in the queue} = [\text{present position of first person} + \text{previous position of second person}] - 1$$



Example: - In a row of girls, Shilpa is eighth from the left and Reena is seventeenth from the right. If they interchange their positions, Shilpa becomes fourteenth from the left. How many girls are there in the row?

Solution:

Total no. of girls

$$= [\text{present position of Shilpa} + \text{previous position of Reena}] - 1$$

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$$= (14 + 17) - 1 = 30$$

Type – 4: Previous position of first person or present position of second Person

Previous position of first person or present position of second Person

= Difference of the two positions of second person + previous position of second person.

OR

= Difference of the two positions of first person + previous position of second person.



Example: - In a row of children, Dipa is fifth from the left and Vijay is sixth from the right. When they interchange their places among themselves, Dipa becomes thirteenth from the left. Then what will be Vijay's position from the right?

Solution:

Present position of Vijay = Difference of the two positions of Dipa + previous position of Vijay

$$= (13 - 5) + 6 = 14^{\text{th}}$$

12.2 Time Sequence Test

Let's tackle some basic time-related questions.



Example: - After every 20 minutes from a bus stand a bus leaves for Jaisalmer. When the passenger reached the bus had left already 10 minutes ago and the next bus will leave at 9:35 AM. At what time passenger reached at the bus stop?

Solution:

We know that the next bus will arrive at 9:35 AM

Hence, the bus departed at 9:15 AM and the passenger came 10 mins after that so, $9:15 + 10 = 9:25 \text{ AM}$. Hence, the passenger reached bus stop at 9:25 AM.



Example: - Satish remembers that his brother's birthday is after fifteenth but before eighteenth of February whereas his sister Kajal remembers that her brother's birthday is after sixteenth but below nineteenth of February. On which day in February is Satish's brother's birthday?

- (A) 16th
- (B) 17th
- (C) 18th
- (D) 19th
- (E) None of these

Solution:

According to Satish, the brother's birthday is on one of the days among 16th and 17th February. According to Kajal, the brother's birthday is on one of the days among 17th and 18th February. Clearly, Satish's brother's birthday is on the day common to both the above groups, i.e., 17th February.

12.3 Problems on Ranking and Time Sequence Test



Example 1: - A class of boys stands in a single line. One boy is nineteenth in order from both the ends. How many boys are there in the class?

- (a) 27
- (b) 37
- (c) 38
- (d) 39

Solution:

(b) Clearly, number of boys in the row = $(18 + 18 + 1) = 37$

The question can be solved by formula also, total number of boys in the row = $19 + 19 - 1 = 37$

Formula

Total number of persons in a row or class = (Rank of a person from upper end or left end) + (Rank of that person from lower or right end) - 1



Example 2: - Rohan ranks seventh from the top and twenty-sixth from the bottom in a class. How many students are there in the class?

- (A) 31
- (B) 32
- (C) 33
- (D) 34

Solution:

Clearly, the whole class consists of: 6 student who have ranks higher than Rohan; and 25 students who have ranks lower than Rohan, i.e, $(6+1+25) = 32$ students. Hence, the answer is (b).



Example 3: - Karishma ranks 10th from the top and 15th from the bottom in an examination. Find the total number of students in Karishma's class.

- (a) 35
- (b) 31
- (c) 28
- (d) 30

Solution:

- (d) As per the question; the class has
- (i) 15 students higher than Karishma
- (ii) 14 students lower than Karishma
- (iii) Karishma itself

∴ Total number of students = $15 + 14 + 1 = 30$ Hence, option (d) is correct.



Example 4: - In a row of trees, one tree is fifth from either end of the row. How many trees are there in the row ?

- A. 8
- B. 9
- C. 10
- D. 11

Solution:

B . 9

Justification: Clearly, number of trees in the row = $(4 + 1 + 4) = 9$.



Example 5: - Neena returned home 3 days earlier than the time she had told her mother. Neena's sister Veena reached five days later than the day Neena was supposed to return. If Neena returned on Thursday, on what day did Veena return?

- (a) Friday
- (b) Saturday
- (c) Wednesday
- (d) Sunday

Solution:

Neena returned home on Thursday. Neena was supposed to return 3 days later, i.e., on Sunday.

Veena returned five days later from Sunday, i.e., on Friday.

∴ Option (a) is the correct option.



Example 6: - If the seventh day of a month is three days sooner than Friday, what day will it be on the nineteenth day of the month.

Solution:

Unit 12: Ranking and Time Sequence Test

Given that seventh day of a month is three days sooner than Friday. So, the seventh day of the month is three days earlier than Friday, which is Tuesday. Therefore, the fourteenth day is also Tuesday. Hence, nineteenth day is Sunday.



Example 7: - Vandana remembers that her father's birthday is between 13th and 16th of June. Whereas her brother remembers that their Father's birthday is between 14th and 18th of June. On which day is their Fathers birthday?

- (a) 14th June
- (b) 16th June
- (c) 15th June
- (d) 18th June

Solution:

According to Vandana her father's birthday is on one of the days among 14th and 15th June. Vandana's brother, the father's birthday is one of the days among 15th 16th and 17th June. It is obvious that the father's birthday is on the day to both the above groups. The common day is 15th. Hence, the father's birthday falls on 15th June.

∴ Option (c) is the correct option.

Summary

The key concepts learned from this unit are: -

- Understood the different types of questions on Ranking Test.
- Understood the different types of questions on Time Sequence Test.

Keywords

- Ranking Test.
- Time Sequence Test.

Self Assessment

1. In a queue, Amrita is 10th from the front while Mukul is 25th from behind and Mamta is just in the middle of the two. If there were 50 persons in the queue, what position does Mamta occupy from the front?

- A. 20th
- B. 19th
- C. 18th
- D. 17th

2. Manisha ranked sixteenth from the top and twenty-ninth from the bottom among those who passed an examination. Six boys did not participate in the competition and five failed in it. How many boys were there in the class?

- A. 40

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- B. 44
- C. 50
- D. 55

3. Some boys are sitting in a row. P is sitting fourteenth from the left and Q is seventh from the right. If there are four boys between P and Q, how many boys are there in the row?

- A. 25
- B. 23
- C. 21
- D. 19

4. A bus for Delhi leaves every thirty minutes from a bus stand. An enquiry clerk told a passenger that the bus had already left ten minutes ago and the next bus will leave at 9.35 a.m. At what time did the enquiry clerk give this information to the passenger?

- A. 9.10 a.m.
- B. 8.55 a.m.
- C. 9.08 p.m.
- D. 9.15 a.m.

5. Sangeeta remembers that her father's birthday was certainly after eighth but before thirteenth of December. Her sister Natasha remembers that their father's birthday was definitely after ninth but before fourteenth of December. On which date of December was their father's birthday?

- A. 10th
- B. 11th
- C. 12th
- D. Data inadequate

6. The train for Lucknow leaves every two and a half hours from New Delhi Railway Station. An announcement was made at the station that the train for Lucknow had left 40 minutes ago and the next train will leave at 18.00 hrs. At what time was the announcement made?

- A. 15.30 hrs
- B. 15.50 hrs
- C. 16.10 hrs
- D. 17.10 hrs

7. Akshay is 16th from the left end in the row of boys and Vijay is 18th from the right end. Avinash is 11th from Akshay towards the right end and 3rd from Vijay towards the right end. How many boys are there in the row?

- A. Data inadequate
- B. 41
- C. 40
- D. 48

8. In a row of children facing North, Ritesh is twelfth from the left end. Sudhir who is twenty-second from the right end is fourth to the right of Ritesh. Total how many children are there in the row?

- A. 35
- B. 36
- C. 37
- D. 34

9. In a row of 40 students facing North, Kailash is 6th to the left of Soman. If Soman is 30th from the left end of the row, how far is Kailash from the right end of the row?

- A. 17th
- B. 16th
- C. 15th
- D. 26th

10. Madhu is 18th from the left end and Sandhu is 11th from the right end of a row of 40 children. How many children are there between Madhu and Sandhu in the row?

- A. 10
- B. 9
- C. 12
- D. 11

11. In a class of 20 students, Alisha's rank is 15th from the top. Manav is 4 ranks above Alisha. What is Manav's rank from the bottom?

- A. 10th
- B. 11th
- C. 9th
- D. 12th

12. Rahul ranked ninth from the top and thirty eighth from the bottom in a class. How many students are there in the class?

- A. 45
- B. 46
- C. 47
- D. 48

13. In a row of boys, Deepak is seventh from the left and Madhav is twelfth from the right. If they interchange their positions, Deepak becomes twenty-second from the left. How many boys are there in the row?

- A. 19
- B. 31

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C. 33

D. Cannot be determined

14. Manish ranked sixteenth from the top and twenty ninth from the bottom among those who passed an examination. Six boys did not participate in the competition and five failed in it. How many boys were there in the class?

A. 40

B. 44

C. 50

D. 55

15. Kailash remembers that his brother Deepak's birthday falls after 20th May but before 28th May, while Geeta remembers that Deepak's birthday falls before 22nd May but after 12th May. On what date Deepak's birthday falls?

A. 20th

B. 21th

C. 22th

D. 23th

Answers for Self Assessment

1. C 2. D 3. A 4. D 5. D

6. C 7. B 8. C 9. A 10. D

11. A 12. B 13. C 14. D 15. B

Review Questions

1. Rohan ranks 7th from the top and 26th from the bottom in the class. How many students are there in the class?

2. Manik is 14th from the right end in the row of 40 students. What is his position from the left end?

3. In a row of boys facing the north, A is 16th from the left end and C is 16th from the right end. B, who is 4th to the right of A, is 5th to the left of C, in a row. How many boys are there in a row?

4. In a row of students, Anil is 7th from left, while Sunil is 18th from right. Both of them interchanged their positions such that Anil becomes 21st from left. What will be the total number of students in the class?

5. Mohan is taller than Shyam but shorter than Ramesh. Ramesh is taller than Rajat but shorter than Gautam. If Shyam is taller than Rajat, then who is the shortest among all?

6. Sunita leaves her house at 20 min to seven in the morning, reaches Vineeta's house in 25 min, they finish their breakfast in another 15 min and leave for their office which takes another 35 min. At what time, did they leave Vineeta's house to reach their office?

7. Standing on a platform, Amit told Sunita that Aligarh was more than ten kilometres but less than fifteen kilometres from there. Sunita knew that it was more than twelve but less than fourteen kilometres from there. If both of them were correct, which of the following could be the distance of Aligarh from the platform?

Unit 12: Ranking and Time Sequence Test

8. Reaching the place of meeting on Tuesday 15 minutes before 08.30 hours, Anuj found himself half an hour earlier than the man who was 40 minutes late. What was the scheduled time of the meeting?
9. The priest told the devotee, "The temple bell is rung at regular intervals of 45 minutes. The last bell was rung five minutes ago. The next bell is due to be rung at 7.45 a.m." At what time did the priest give this information to the devotee?
10. If the seventh day of a month is three days earlier than Friday, what day will it be on the nineteenth day of the month?

**Further Reading**

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

**Web Links**

1. <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
2. <https://www.hitbullseye.com/Reasoning>

Unit 13: Direction Sense Test

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Sense direction correctly.
- Understand and solve distance related questions.
- Understand left right movement.
- Understand and solve shadow type of questions of directions test.
- Understand and solve puzzle type of direction sense test.

Introduction

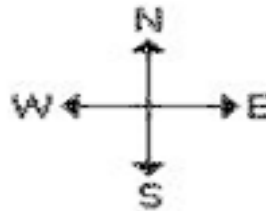
The questions based on directions require the candidates to identifying the direction of an individual or shadow from a set of statements. We should know of some of the important points which helps to solve the questions with ease.

13.1 General Directions

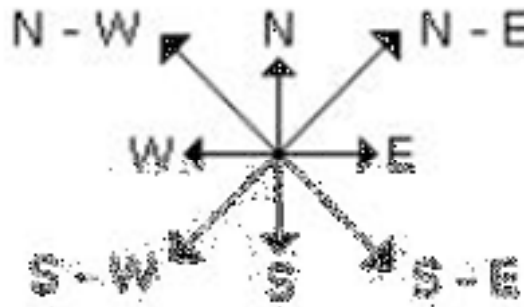
In general, there are four main directions i.e., North, South, East and West. Apart from these four, there are four additional directions derived from the main ones. They are called North-East, North-West, South-East and South-West. A chart is given below for reference.



There are four main directions – North, South, East and West.



There are four cardinal directions – North-East(N-E), North-West(N-W), South-East(S-E), South-West(S-W).



13.2 Casting of Shadows

A lot of times questions in CAT are asked regarding casting of shadows. It is important and useful to know the below concepts to understand.

- At the time of sunrise if a man stands facing the east, his shadow will be towards west.
- At the time of sunset, the shadow of an object is always in the east.
- If a man stands facing the North, at the time of sunrise his shadow will be towards his left and at the time of sunset it will be towards his right.
- At 12.00 noon, the rays of the sun are vertically downward hence there will be no shadow.
- Always rainbow will occur opposite to the sun.
- Always shortest distance should be calculated.

13.3 Distance Related Questions

In most of the direction and distance questions, it is required to calculate a certain parameter from the question statement. The questions can be related to the total distance walked, shortest path, the distance between two entities, etc.

While solving the distance related question, one must be thorough with the Pythagorean Theorem to be able to solve most of the questions. The Pythagorean Theorem is used to calculate the shortest path travelled, the minimum distance between two points, etc.

13.4 Left Right Movement

A person facing north, on taking left will face towards west and on taking the right turn towards east. A person facing west, on taking left will face towards south and on taking right turn towards north. A person facing east, on taking left will face towards north and on taking the right turn towards south. A person facing south, on taking left will face towards east and on taking the right turn towards west.

13.5 Direction Puzzles

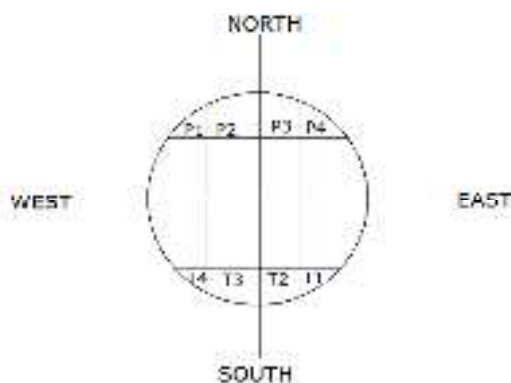
In the Direction Sense Test, the questions consist of a sort of direction puzzle. A successive follow-up of directions is formulated and the candidate is required to ascertain the final direction or the distance between two points. The test is meant to judge the candidate's ability to trace and follow correctly and sense the direction correctly.



Example:

Directions for Q1 to Q4:

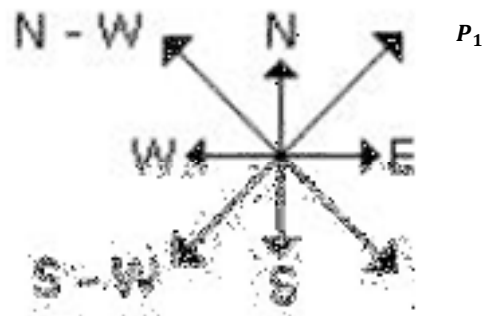
Four Policemen P1, P2, P3 and P4 are combing a circular park which is divided into 16 plots. The thieves T1, T2, T3 and T4 who are in the park are to be arrested. The figure below shows their position.



Q.1. From the given alternatives, identify the pair where one is positioned in the North-West and the other in the South-East?

- A. P2, T3
- B. P1, T1
- C. T4, P4
- D. T2, P3

Ans: B P1, T1

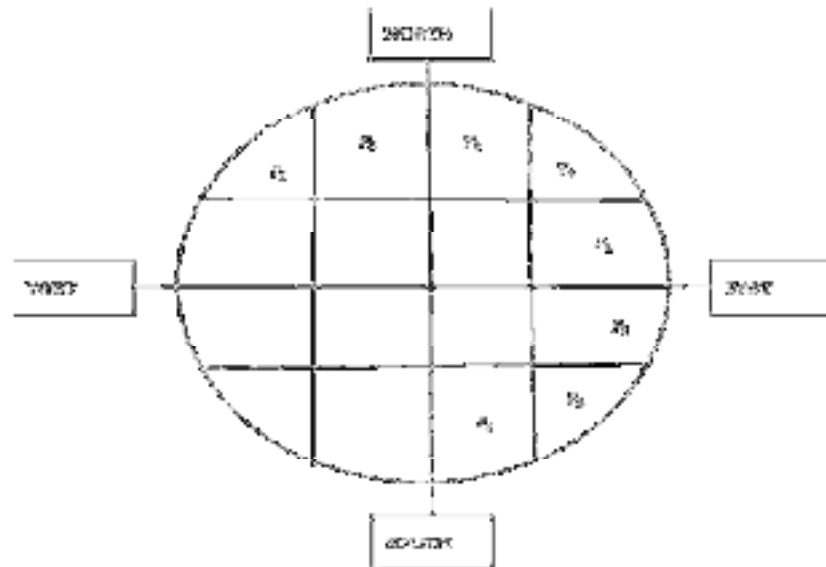
T₁

Q.2. If P₁, P₂, P₃, P₄ move four plots in a clockwise direction and T₁, T₂, T₃, T₄ move six plots in an anti-clockwise direction, who would be in the North-South direction?

- A. P₂, T₂
- B. T₃, P₄
- C. T₄, P₁
- D. T₁, P₃

Ans: B T₃, P₄

After moving the new positions of four police men and four thieves are shown by below chart.

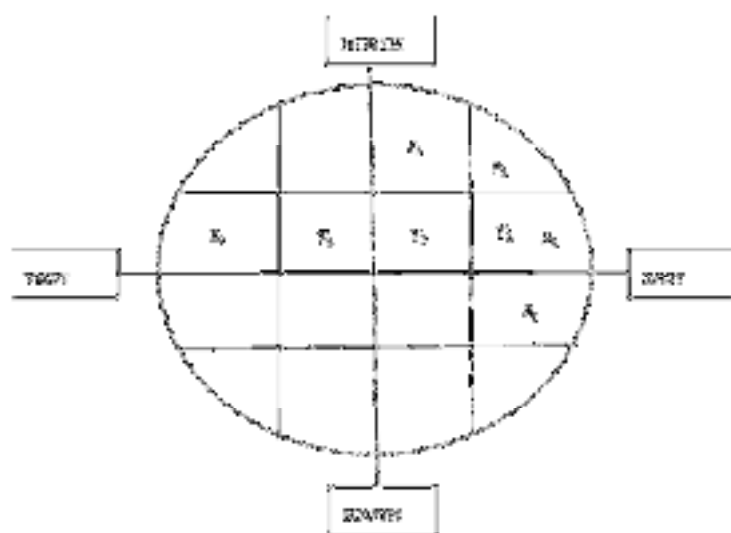


Q.3. If the four policemen move two plots clockwise and T₁, T₂, T₃, T₄ move two plots vertically up, who will be able to catch whom?

- A. P₃, T₁
- B. P₄, T₂
- C. P₁, T₃
- D. P₂, T₄

Ans: A. P₃, T₁

After moving the new positions of four police men and four thieves are shown by below chart.

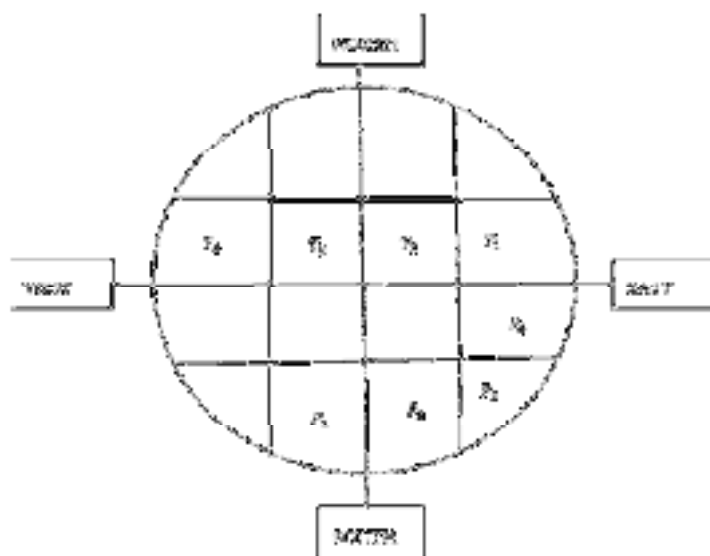


Q.4. If the policemen move four plots anti-clockwise and the criminals move two plots vertically up, who would be facing North-Southwards?

- A. P2, T1
- B. P1, T3
- C. P1, T4
- D. P4, T4

Ans: B P1, T3

After moving the new positions of four police men and four thieves are shown by below chart.



13.6 Problems on Direction Sense Test



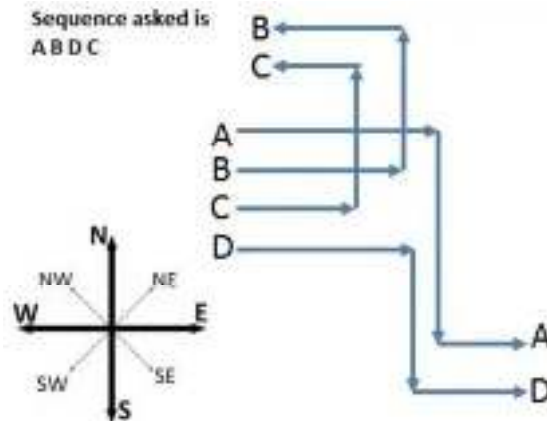
Example: 1 - Airplanes A, B, C and D started flight towards east. After flying 125 kms planes A and D flew towards right while planes B and C flew towards left. After 115 km, planes B and C flew towards their left while planes A and D also turned towards their left. In which directions are the airplanes A, B, D, C respectively flying now?

- A. North, South, East, West
- B. East, West, West, East
- C. East, West, East, West
- D. South, North, North, South

Analytical Skills

Solution: - Let us look at A and D first. After flying for 125 kms, A and D would still be in east direction. Now, they take a right turn which makes their direction as south. From south, if you take a left, you will again come in the east direction. So, planes A and D, are in East direction.

Now, let us look at planes B and C. After flying for 125 kms, B and C would still be in east direction. From east direction, they take a left turn which means now they are in north. From north, they again take a left which makes them land in the west.

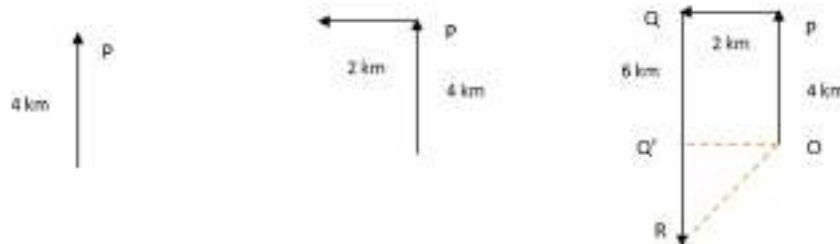


So correct answer would be: East, west, East, West (Option C)



Example: 2 - A person starts walking towards North for about 4 km and reaches a point P. From point P, he takes a left turn and walks for 2 km to reach a point Q. From Q, he again takes a left turn and walks for 6 km and reaches a point R. How far and in which direction is the point R from the starting point?

Solution: Let us first draw a rough sketch of the above reasoning direction problem.



Joining the starting point O to the endpoint R and drawing a perpendicular on QR at Q'.

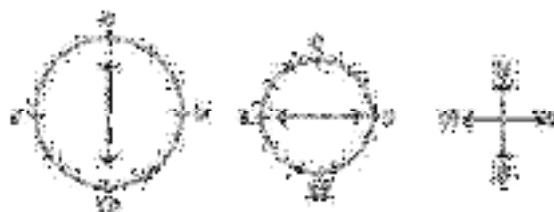
In a right-angled triangle OQ'R, we have OQ' = PQ = 2 km, and Q'R = QR - QQ' = 6 - 4 = 2 km.

So, $OR^2 = 2^2 + 2^2 = 4 + 4 = 8$. or, $OR = \sqrt{8} = 2\sqrt{2}$ km. R is in the South-West direction from the starting point O.



Example: 3 - Rahul put his timepiece on the table in such a way that at 6 P.M. hour hand points to North. In which direction the minute hand will point at 9.15 P.M.?

Solution:



At 9.15 P.M., the minute hand will point towards west.



Example: 4 - Town A is to the West of Town B. Town P is to the South of Town A. Town Q is to the East of Town P. Then Town Q is towards which direction of Town A?

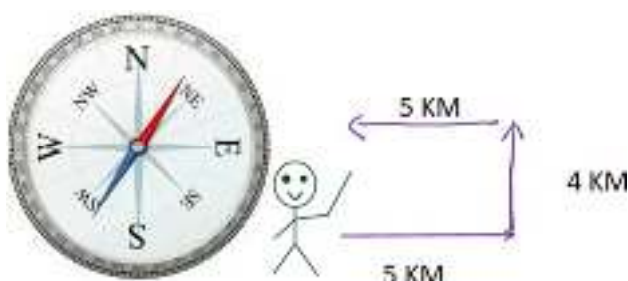
- A. South
- B. South West
- C. South East
- D. North

Solution: - C. South East



Example: 5 - Siva starting from his house, goes 5 km in the East, then he turns to his left and goes 4 km. Finally, he turns to his left and goes 5 km. Now how far is he from his house and in what direction?

Solution: 4Km North



Example: 6 - If North-east becomes West and South-east becomes North then what will West become?

- A. South-east
- B. North-east
- C. South
- D. North-west
- E. None of these

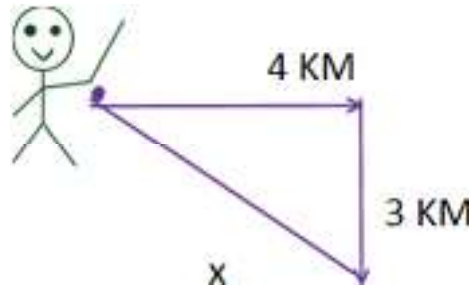
Solution: Since all directions are moving anticlockwise with 145° . Therefore, West becomes South-east as shown in below two figures.





Example: 7 - Suresh starting from his house, goes 4 km in the East, then he turns to his right and goes 3 km. What minimum distance will be covered by him to come back to his house?

Solution:



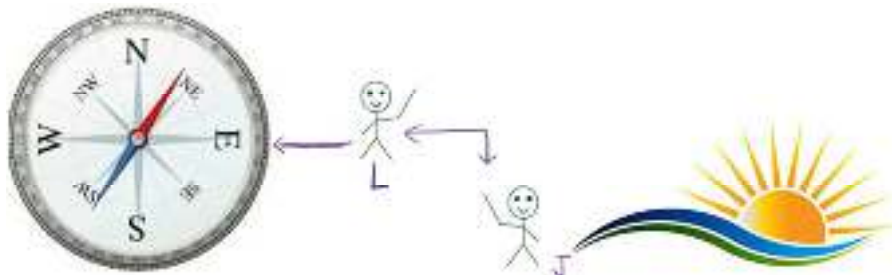
$$X^2 = 4^2 + 3^2 = 25$$

$$\text{Or, } X = 5 \text{ km}$$



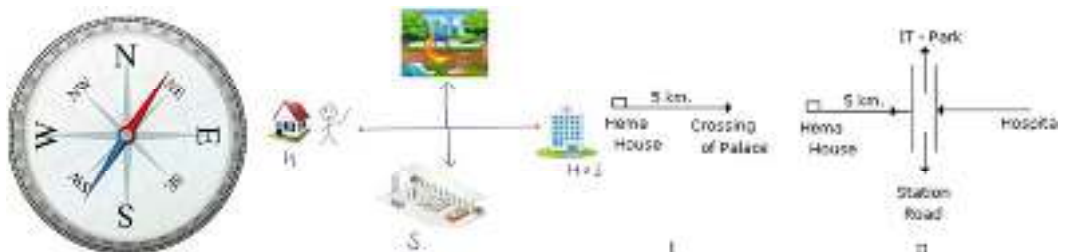
Example: 8 - One morning after sunrise Juhi while going to school met Lalli at Boring Road crossing. Lalli's shadow was exactly to the right of Juhi. If they were face to face, which direction was Juhi facing?

Solution: The sun rises in the East (E) in the morning and shadow will be in the West. As the shadow of Lalli falls to his right. So, Juhi must be facing South.



Example: 9 - Hema starting from her house walked 5 km to reach the crossing of Palace. In which direction she was going, a road opposite to this direction goes to Hospital. The road to the right goes to station. If the road which goes to station is just opposite to the road which IT-Park, then in which direction to Hema is the road which goes to IT-Park?

Solution:



It is clear that the road which goes to IT-Park is left to Hema.



Example: 10 - One morning after sunrise, Mahesh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?

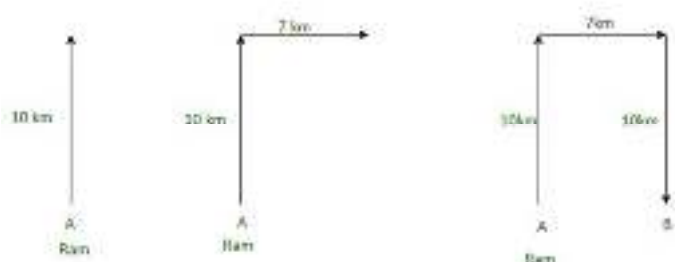
Solution: -

The sun rises in the East (E) in the morning and shadow will be in the West. As the shadow of pole falls to his right. So, he must be facing South. Hence, the answer is the south.



Example: 11- Ram starts from a point A walks 10 km north, then turns right and walks for 7 km, then turns right again and walks for another 10 km. And reaches point B. How far is Ram from the starting point?

Solution: - 7 km as shown below



Example: 12

One evening, two friends Riya and Priya were talking to each other, with their backs towards each other, sitting in a park. If Riya's shadow was exactly to the left of her, then which direction was Priya facing?

Solution:

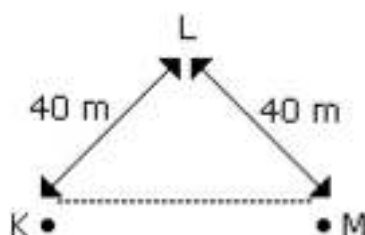
Riya's shadow fell to her left i.e., towards East (as it was evening). So, Riya was facing "South". As Priya had her back towards Riya, hence, Priya was facing "North". North is the correct answer.



Example: 13

K is 40 m South-West of L. If M is 40 m South-East of L, then M is in which direction of K?

Solution: M is in the East of K as shown in the following diagram. East is the correct answer.





Example: 14 -While facing East you turn to your left and walk 10 yards; then turn to your left and walk 10 yards: and now you turn 45° towards your right and go straight to cover 25 yards. Now, in which direction are you from your starting point?

- A. North-East
- B. South-West
- C. East
- D. North-West
- E. None of these

Solution: - D. North-West

Let be the initial position of the man. He initially faced East and then turned his left in the direction of North and walked 10 yards. Then, turned his left in the direction of West and walked 10 yards. Now, he turned 45° to his right and walked 50 yards straight in the same direction. Now, the direction of the man with respect to his starting point is North - West.



Example: 15 -A direction pole was situated on the crossing. Due to an accident the pole turned in such a manner that the pointer which was showing East, started showing South. One traveller went to the wrong direction thinking it to be West. In what direction actually he was travelling?

- A. South
- B. East
- C. West
- D. North
- E. None of these

Solution: - D. (North)

Due to an accident, the pointer showing East, started showing South. It means, the pole has been rotated through 90° clockwise. So, when she was travelling towards West, actually it was North.



Example: 16 -My friend is facing east. She turned 120° in the clockwise direction and then 165° in the anti-clockwise direction. Which direction is she facing now?

- A. East.
- B. North-east.
- C. North.
- D. South-west.
- E. None of these

Answer: - B. North-east.

$(165^\circ - 120^\circ = 45^\circ)$ 45° to anticlockwise from the initial directions i.e., North-East.



Example: 17 -Five friends are playing a game. Nitin is facing South. He follows instructions given by his friends.

- (i) When Kimi touch him, Nitin walks 50 metres.
- (ii) When Neha touches him, Nitin turns right and walks 40 metres.
- (iii) When Annu touch him, Nitin turns left and walks 80 metres.
- (iv) When Punit touches him, Nitin walks 70 metre in the direction opposite to which he faces.

If Kimi, Annu, Punit and Neha touch him in that order, how far does Nitin reach from his starting point?

- A. 60 metres.
- B. 72 metre.
- C. 80 metre.
- D. 75 metre.
- E. None of these

Solution: - B. 72 metre.



Example: 18 - Read the situations given below to answer the questions:

Nine cars P, Q, R, S, T, U, V, W and X are parked such that:

- (1) R is 4 km east of Q.
- (2) P is 2 km north of Q.
- (3) W is 4 km south of P.
- 4) V is 2 km west of W.
- (5) S is 6 km east of V.
- (6) U is 4 km north of V.
- (7) X is parked in the middle of Q and R.
- (8) T is parked in the middle of W and S.

What is the distance between W and S?

- A. 6 km.
- B. 4 km.
- C. 3 km.
- D. 2 km.
- E. 5 km

Solution: - B. 4 km.



Example: 19 -What is the direction of Q with respect to W?

- A. East
- B. West
- C. South

Analytical Skills

- D. North
- E. Canno

Solution: - D. North



Example: 20 - What is the distance between U and W?

- A. 20 km
- B. $\sqrt{28}$ km
- C. $\sqrt{20}$ km
- D. $3\frac{4}{5}$ km
- E. Cannot be determined

Solution: - C. $\sqrt{20}$ km



Example: 21 - Which car is parked between V and T?

- A. S
- B. Q
- C. W
- D. T
- E. Can't be determined

Solution: - C. W



Example: - DIRECTIONS (22-24) Read the following information carefully and answer the questions given below it:

- (A) $P \alpha Q$ means Q is to the right of P at a distance of one metre.
- (B) $P \beta Q$ means Q is to the North of P at a distance of one metre.
- (C) $P \lambda Q$ means Q is to the left of P at a distance of one metre.
- (D) $P \eta Q$ means Q is to the South of P at a distance of one metre.
- (E) In each of the following questions all person's face South.



Example: 22 - If $A \eta B \lambda L \beta K$, then K is in which direction with respect to A?

- A. South
- B. East
- C. North
- D. West
- E. None of these

Solution: - B. East



Example: 23 - If $G \alpha L \eta R \alpha M$ then M is in which direction with respect to L?

- A. North-east
- B. North-west
- C. South-east
- D. South-west
- E. None of these

Solution: - D. South-west



Example: 24 - If $A \alpha B \lambda C \beta D$, then D is in which direction with respect to A?

- A. North
- B. South
- C. East
- D. North-east
- E. None of these

Solution: - A. North



Example: 25 - Facing to the South Mr. Jha turns in certain ways, which of the following turns will not lead him to the same side?

- A. left, left, right, left, left, right
- B. left, left, left, right, right, right
- C. left, right, left, right, left, right
- D. right, left, right, left, right, left
- E. None of these

Solution: - A. left, left, right, left, left, right

Summary

The key concepts learned from this unit are: -

- Understood and solved distance related questions.
- Understood left right movement.
- Understood and solved shadow type of questions of directions test.
- Understood and solved puzzle type of direction sense test.

Keywords

- Distance
- Directions
- East
- West
- South
- North
- South-East
- North-East
- South-West
- North-South

Self Assessment

1. If North-east becomes West and South-east becomes North then what will West become?
 - A. South-east
 - B. North-east
 - C. South
 - D. North-west

2. A man walks 6 km towards the north, then turns towards his left and walks for 4 km. He again turns left and walks for 6 km. At this point he turns to his right and walks for 6 km. How many km and in what direction is he from the starting point?
 - A. 10 km, West
 - B. 6 km, South
 - C. 4 km, South
 - D. 8 km, West

3. A river flows west to east and on the way turns left and goes in a semi-circle round a hillock and then turns left at right angles. In which direction is the river finally flowing?
 - A. West
 - B. East
 - C. North
 - D. South

4. While facing East you turn to your left and walk 10 yards; then turn to your left and walk 10 yards: and now you turn 45o towards your right and go straight to cover 25 yards. Now, in which direction are you from your starting point?
 - A. North-East
 - B. South-West
 - C. East
 - D. North-West

5. A direction pole was situated on the crossing. Due to an accident the pole turned in such a manner that the pointer which was showing East, started showing South. One traveller went to the wrong direction thinking it to be West. In what direction actually he was travelling?
- A. South
B. East
C. West
D. North
6. Five friends are playing a game. Nitin is facing South. He follows instructions given by his friends.
- (i) When Kimi touch him, Nitin walks 50 metres.
(ii) When Neha touches him, Nitin turns right and walks 40 metres.
(iii) When Annu touch him, Nitin turns left and walks 80 metres.
(iv) When Punit touches him, Nitin walks 70 metre in the direction opposite to which he faces.
- If Kimi, Annu, Punit and Neha touch him in that order, how far does Nitin reach from his starting point?
- A. 60 metres.
B. 72 metres.
C. 80 metres.
D. 75 metres.
7. Pradeep starts walking from his college towards his house. He starts from the front gate of his college and walks 5 km, then turns left and walks 2 km, then turns left again and walks 4 km, then he turns to his right and walks 3 km, then turns left and walks 1 km and then turns to his left again and walks 4 km, then turns to his right and walks 10 km and finally turns right and walks 3 km and thus reaches the front gate of his house. If Pradeep's house is facing south, in which direction did he start walking?
- A. West
B. North
C. South
D. East
8. Ram is facing East. Turning to the right he went for 20 m, then turning to the left he went 20 m and turning to the right and went 20 m, then again turning to the right went 40 m and then again, went for 40 m to the right. In which direction Ram is moving from his original position?
- A. North
B. West
C. South
D. East

Analytical Skills

9. Bhairav walked 30 ft towards North, then took a left turn and walked 15 ft. He again took a left turn and walked 30 ft. How far and in which direction is Bhairav from the starting point?
- A. 15 ft to the West
 - B. 45 ft to the South
 - C. 30 ft to the East
 - D. 15 ft to the North
10. Seema started early in the morning on the road towards the Sun. After some time, she turned to her left. Again, after some time she turned to her right. After moving some distance, she again, turned to her right and began to move. At this time, in what direction was she moving?
- A. South
 - B. North-West
 - C. North-East
 - D. East
11. At dusk, Rohit started walking facing the sun. After a while, he met his friend and both turned to their left. They halted for a while and started moving by turning again to their right. Finally, Rohit waved 'good bye' to his friend and took a left turn at a corner. At which direction is Rohit moving now?
- A. South
 - B. West
 - C. North
 - D. East
12. Sumi ran a distance of 40 m towards South. She then turned to the right and ran for about 15 m, turned right again and ran 50 m. Turning to right then ran for 15 m. Finally, she turned to the left an angle of 45° and ran. In which direction was she running finally?
- A. South-East
 - B. South-West
 - C. North-East
 - D. North-West
13. A man starting from his home moves 4 km towards East, then he turns right and moves 3 km. Now what will be the minimum distance covered by him to come back to his home in km?
- A. 4
 - B. 5
 - C. 12
 - D. 13

14. Pran and Khan start from their office and walks in opposite direction, each traveling 10 km. Pran then turns left and walks 10 km. While Khan turns right and walks 10 km. How far they are now from each other?
- A. 0 km
B. 5 km
C. 10 km
D. 20 km
15. One morning after sunrise, Vikram and Shailesh were standing in a town with their backs towards each other. Vikram's shadow fell exactly towards left and side. Which direction was Shailesh facing?
- A. East
B. West
C. North
D. South

Answers for Self Assessment

1.	A	2.	A	3.	B	4.	D	5.	A
6.	B	7.	D	8.	B	9.	D	10.	A
11.	A	12.	D	13.	B	14.	D	15.	D

Review Questions

- Satish starts from his house and takes two right turns and then one left turn. Now he is moving towards south. In which direction Satish started from his house?
- A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now?
- Sumi ran a distance of 40 m towards South. She then turned to the right and ran for about 15 m, turned right again and ran 50 m. Turning to right then ran for 15 m. Finally, she turned to the left an angle of 45° and ran. In which direction was she running finally?
- A child is looking for his father. He went 90 metres in the east before turning to his right. He went 20 metres before turning to his right again to look for his father at his uncle's place 30 metres from this point. His father was not there. From there, he went 100 metres to his north before meeting his father in a street. How far did the son meet his father from the starting point?
- Deepa moved a distance of 75 metres towards the north. She then turned to the left and walking for about 25 metres, turned left again and walked 80 metres. Finally, she turned to the right at an angle of 45° . In which direction was she moving finally?
- From his house, Rahul went 25 km to north. Then he turned towards west and covered 15 km. Then he turned south and covered 10 km. Finally, turning to east, he covered 15 km. In which direction is he from his house?

Analytical Skills

7. Rasik walked 20 m towards north. Then he turned right and walks 30 m. Then he turns right and walks 35 m. Then he turns left and walks 15 m. Finally, he turns left and walks 15m. In which direction and how many metres is he from the starting position?
8. One morning after sunrise, Suresh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?
9. One morning Udai and Vishal were talking to each other face to face at a crossing. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?
10. Ranuka started walking from her house, she first walked for 3 km towards west, then she turned towards north and moved 4 km in that direction. How far is Renuka from her house?



Further Reading

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle



Web Links

1. <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
2. <https://www.hitbullseye.com/Reasoning>

Unit 14: Blood Relation

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Objectives

The key objective of this unit for a learner is expected to learn and master of the following topics after going through the unit: -

- Understand how to find Family or Blood Relationship.
- Understand how to find Relationships Involving the Term „-in-law“ General.
- Understand how to find Relationships Involving Father-in-law, Mother-in-law, Son-in law and Daughter-in-law
- Understand how to find Relationships Involving Brother-in-law and Sister-in-law
- Understand how to find Relationships Half Sibling and Step Relations.

Introduction

Blood Relations is the most studied topic of Logical Reasoning and is one of the few topics which has found its importance in almost every entrance exam. This topic tests the analytical skills of the students and how one can approach the solution of logical problems with the help of diagrams instead of calculations. The diagrams play an important role here. The right diagrams come from the right analysis. The following is the list of exams in which the appearance of questions from blood relations are frequent. Any relation in the world which either by birth or by marriage is called a Blood Relation. Relation as we know is the connection between any two things or between any two persons. So, when speaking of blood relation, it means the connection between two people by blood or basically by birth.



Example: Any relation by birth will be mother, father, son, daughter, etc. and any relation by marriage will be father-in-law, mother-in-law, etc.

14.1 Types of Questions on Blood Relation

Over the years, the standard and type of questions which are being asked from the blood relation topic have seen a slight turn. Initially, the questions used to be less complex and statement or dialogue-based, but with the increased competition, the variety of questions being asked have also changed.

Being one of the most common concepts from which questions are asked in exams, given below are the different ways in which the blood relation questions may be asked in the competitive exams, for the assistance of candidates:

Dialogue/ Conversation Based – Based on Dialogue or conversation. In such questions, one person describes his/her relation with another person (this may or may not be related to the person with whom the conversation is being made).

Based on Puzzles – To make the questions complex, blood relation questions are also being asked in the form of a puzzle. A piece of brief information about multiple people being interrelated is given and sub-questions based on the same may be asked.

Coding-Decoding – The relationship between two people may be denoted using symbols. This has become a common method of asking blood relation questions in competitive exams, nowadays.

Keep in mind that from a person's name, we cannot judge the gender of that person. The name doesn't always show the gender beyond a reasonable doubt.

A tree as we know has roots firmly grounded and then it has its stem which branches out and gets leaves in it.

Basic Terms used in Blood Relations

Parents: Mother and father.

Children: Son or Daughter

Siblings: Brother or Sister (of the same parents)

Spouse: Husband or wife.

Aunt: Aunt means father's sister, mother's sister, father's brother's wife, or mother's brother's wife.

Uncle: Uncle means father's brother, Mother's brother, Father's sister's husband, mother's sister's husband.

Niece: Brothers and sister's daughter.

Nephew: Brother's and sister's son.

Cousin: Children of aunt and uncle.

Father-in-law: Father of the spouse.

Mother-in-law: Mother of Spouse.

Sister-in-law: Sister of the spouse, wife of brother

Co-sister: Wife of spouse's brother.

Brother-in-law: Brother of the spouse, husband of the sister

Co- brother: husband of spouse's sister.

Maternal: Relations or family members who are from the mother's side.

Paternal: Relations or family members who are from the father's side.



Note:

I. Any relation of Mother's side is called „Maternal“.

II. Any relation of Father's side is called „Paternal“.



For Example: Pointing towards a boy Veena said “He is the son of only son of my grandfather”. How is that boy related to Veena?

Solution: He is the son of only son of my grandfather – Veen’s Father’s son. Therefore, that boy is a Veena’s brother.

The same is the case with a family tree, wherein we can imagine our ancestors as the roots and then their children and grandchildren and so on act as branches and leaves.

Family Tree a pictorial or visual representation of our lineage. With the help of a family tree, it not only gives us a better understanding of our lineage but also helps us understand our relationship with different people who have common ancestors.

There also are a few things which need to be kept in mind while solving the blood relation questions. Given below are few such important pointers:

You cannot assume the gender of the person based on the name

If the statement says X is the son of Y, the gender of Y cannot be determined unless mentioned in the question

In puzzle-based questions, a web of relations can be formed, so do not solve such questions in a haste

These questions are scoring and easy to solve, so do not panic if the question seems lengthy

In case of coding-decoding blood relation, use a pictorial description to solve the question. This will make the symbols and relation more clear

Let us now move on to solving a few sample questions to get a better understanding of the concept.

14.2 Family or Blood Relations Test

Tips on finding a relationship between two members of a family:

Step 1: The first and foremost step is to choose the two persons among whom the relationship is to be established.

Step 2: Now once you have chosen the two persons, pin-point the intermediate relationship between two persons, i.e., such relationship through which long drawn relationship can be established between the required persons.

Step 3: In the end, conclude the relationship between two required persons.

Make sure that you represent the gender of a female with a "□" sign and that of a male with a "⊕"

Definition and Concept

Family or Blood Relationship means persons connected by relations like – father-mother, son-daughter, brother-sister, grandfather-grandmother, uncle-aunty, nephew-niece, brother-in law, sister-in-law etc. The list can go on and on adding members from father’s side and mother’s side etc.

Questions in Test of Reasoning on Family /Blood Relationship are about the relationship of a particular person with another person of the family, based on the chain of relationships between other members of that family.

The questions depict relationships among the various members of a family in a roundabout chain. The candidate is expected to find the relation of two particular persons mentioned in the question.

An example of a question on Blood Relationship is given below to understand the concept in a better way:



Example 1: Introducing Neeta, Anil said, „She is wife of my mother’s only son. “ How is Neeta related to Anil?

(1) Mother

- (2) Wife
- (3) Sister
- (4) Daughter-in-law
- (5) None of these

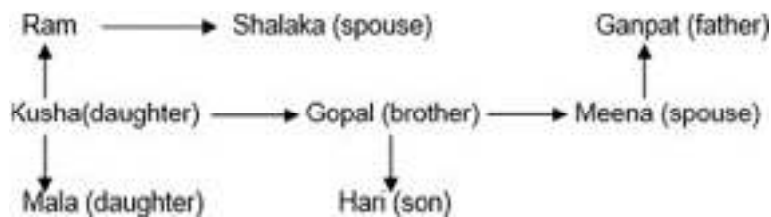
Solution: Neeta is the wife of Anil's mother's only son, who is Anil himself. Hence, answer is Neeta is Anil's wife. i.e. (2) Wife.



Example 2: „Ram“ is the father of „Kusha“ but „Kusha“ is not his son. „Mala“ is the daughter of „Kusha“. „Shalaka“ is the spouse of „Ram“. „Gopal“ is the brother of „Kusha“. „Hari“ is the son of „Gopal“. „Meena“ is the spouse of „Gopal“. „Ganpat“ is the father of „Meena“. Who is the granddaughter of „Ram“?

- (1) Hari
- (2) Mala
- (3) Meena
- (4) Shalaka
- (5) None of these

Solution:



„Mala“ is the daughter of „Kusha“ and „Ram“ is the father of „Kusha“. So, „Mala“ is the granddaughter of „Ram“. Hence, answer is (2) Mala.

Family/Blood Relation Tests are an exercise to test the candidate's ability to comprehend and come to the crux of an issue from complex, lengthy and unclear data.

On a lighter note, this topic of Family/Blood Relations should be of interest to the candidates who are fans of Hindi Cinema, as the nature of the questions on Family/Blood Relations are of the type „Hum Aapke Hai Kaun

14.3 Family/Blood Relations Described General

Family/Blood Relations tests largely depend on the candidate's knowledge of family relations. Various family relationships are described below to help the candidates to understand the relationships better and to attempt the questions based on them with confidence.

Some Common Terms

Meaning of some terms often used in questions on family relationship are given below:

- a) Parent – Mother or father
- b) Child – Son or daughter (even if an adult)

- c) Sibling – Brother or sister (Including half-brother and half-sister - one parent in common)
- d) Spouse – Husband or wife

Basic Relationships Aunt, Uncle, Niece and Nephew

Most English speakers use “uncle” for any of four relationships: father’s brother, mother’s brother, father’s sister’s husband, or mother’s sister’s husband.

Again, “aunt” in English could mean father’s sister, mother’s sister, father’s brother’s wife, or mother’s brother’s wife. Brother’s or sister’s son is called nephew. Brother’s or sister’s daughter is called niece. Children of aunt or uncle are called cousins.

14.4 Relationships Involving the Term ‘-in-law’ General

Any relationship term ending with -in-law indicates that the relationship is by marriage and not by blood. In other words, -in-law will be a blood relative of the spouse. 13 In-law relationship terms are always written with hyphens. And the plural is formed on the part before the “-in-law”; for example, “brothers-in-law” and not “brothers-in-law”. The only exception is the general term “in-laws”, which is always plural.

14.5 Father-in-law, Mother-in-law, Son-in-law and Daughter-in-law

Father-in-law is the father of spouse; mother-in-law is the mother of spouse. If parents get divorced and remarry, their new spouses are called stepparents, not mother-in-law and father-in-law. The husband of daughter is son-in-law; the wife of son is daughter-in-law. If spouse has children from a previous marriage, those are called stepchildren, not sons-in-law or daughters-in-law. The person is their stepfather or stepmother, not their father-in-law or mother-in-law.

14.6 Brother-in-law and Sister-in-law

Brother-in-law” and “Sister-in-law” each have two or three meanings as follows:

- a) Sister-in-law could be
 - i) The sister of spouse, or
 - ii) The wife of brother, or
 - iii) The wife of spouse’s brother.
- b) Similarly, Brother-in-law could be
 - i) The brother of spouse, or
 - ii) The husband of sister, or
 - iii) The husband of spouse’s sister

Relationships Involving the Terms ‘Grand’ and ‘Great’

The relationships of the second generation are prefixed with the word Grand. For example, for a person, the first generation below him/her would be that of his/her child/children. The next/second generation would be the children of the children who would be called Grand Children of that person. The next/ third generation children would be called Great Grand Children of that person. This also applies to Niece and Nephew. For example, Son of nephew of a person is called Grand Nephew and so on

Similarly, for a person, the first generation above him would be that of his/her parents (Father/Mother). The next/second generation above him/her would be the parents of the parents who would be called Grand Parents/ Grand Father/ Grand Mother of that person. The next/ third

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generation parents would be called Great Grand Parents/ Great Grand Father/ Great Grand Mother of that person.

This also applies to the collateral relationships. For example, Son of nephew of a person is called Grand Nephew; Brother of Grand Father is called Grand Uncle and so on.

The fourth-generation relationships are called Great Great Grand. For example, Son of Great Grand Son is Great Great Grand Son.

14.7 Half Sibling and Step Relations

Questions on Half Sibling and Step Relations are not very common in Bank exams. The information given below is only for very discerning candidates.

A half sibling (half-brother or half-sister) is a sibling with one shared biological parent. When a parent remarries, the new spouse is the stepfather or stepmother of any children from the previous marriage. The children from a previous marriage are stepsons and stepdaughters. One is called stepbrother or stepsister if they have no parents in common but their parents have married each other. There is two ways Martha could have a stepsister:

a) If Martha's mother marries second time, and her new husband (Martha's new stepfather) already has a daughter from a previous marriage, that daughter is Martha's stepsister because one of her parents is married to one of Martha's parents.

b) If Martha's father marries second time, and his new wife already has a daughter, that daughter is again Martha's stepsister.

A similar rule gives the two ways for stepbrother.

Summary of some common Relationships is given below in tabular forms:

Relation	Commonly Used Terms
Grandfather's or Grandmother's only son	Father
Grandfather's or Grandmother's only daughter-in-law	Mother
Father's father or Mother's	Grandfather
Father's Mother or Mother's	Grandmother
Father's brother or Mother's	Uncle
Father's sister or Mother's	Aunt
Son's wife	Daughter-in-law
Daughter's husband	Son-in-law
Husband's or wife's sister	Sister-in-law
Husband's or wife's brother	Brother-in-law
Brother's wife	Sister-in-law
Brother's or sister's son	Nephew
Brother's or sister's daughter	Niece
Uncle's or aunt's son or daughter	Cousin

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Sister's husband	Brother-in-law
Brother's wife	Sister-in-law
Grand son's or grand daughter's daughter	Grand Grand Daughter

	0	1	2	3	4
0	Common Ancestor (Husband-Wife)	Child (son or daughter)	Grandchild	Great Grandchild	2 Great Grandchild
1	Child (son or daughter)	Sibling (brother or sister)	Nephew or Niece	Grand Nephew or Niece	Great Grand Nephew or Niece
2	Grandchild	Nephew or Niece	First Cousin		
3	Great Grandchild	Grand Nephew or Niece		Second Cousin	
4	2 Great Grandchild	Great Grand Nephew or Niece			Third Cousin

Types of Questions on Blood Relations

Questions on Blood Relations are of the following types:

- Mixed-Up Relationship Descriptions.
- Relationships Riddle.
- Coded Relations

Type I - Mixed-Up Relationship Descriptions**Concept and Example**

In questions of Mixed-Up Relationship Descriptions, a cluttered and roundabout description of relationships is given. The candidate is required to decipher the whole chain of relations and identify the direct/ actual relationship between the concerned persons. A solved example of the Mixed-Up Relationship Descriptions is given below to understand the concept and questions based on it.

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Example: Pointing to a gentleman, Dinesh said “His only brother is the father of my daughter’s father.” How is the gentleman related to Dinesh?

- (1) Uncle
- (2) Grandfather
- (3) Father
- (4) Brother- in-law
- (5) None of these

Solution: The gentleman’s only brother is the father of Dinesh (Dinesh’s daughter’s father is Dinesh himself.). Gentleman is brother of Dinesh’s father. Gentleman is Dinesh’s uncle. Hence, answer is (1) Uncle.

Basic Skills and Tips for Solving Questions on Mixed-Up Relationship Descriptions

To make the chain of relationships clear, where necessary a rough sketch of family tree may be prepared in pencil on question paper on the basis of descriptions given in the question. The person’s of same generation may be placed on same horizontal level and that of different generation one below the other. It may sometimes be necessary to draw two different diagrams and then put them together based on the link provided.

The relationship may be shown by drawing lines / arrows. Short forms as follows may be used to indicate the nature of relationships:

g – Gentleman/ Male	s – Sister	u – Uncle	snl – Son-in-law
l – Lady/ Female	b – Brother	a – Aunt	dl – Daughter-in-law
sp -Spouse	sn – Son	cb – Cousin Brother	pu – Paternal Uncle
ch – Child	d – Daughter	cs – Cousin Sister	mu – Maternal Uncle
h – Husband	gf – Grandfather	nf – Nephew	
w - Wife	gm – Grandmother	nc – Niece	
m – Mother	gs – Grandson	bl – Brother-in-law	
f – Father	gd – Granddaughter	sl – Sister-in-law	

Use of Small letters is suggested to indicate the nature of relationships to avoid confusion of Capital alphabets used in the questions like „A is mother of C”, „D is sister of B’s husband”

Sometimes even re-writing the given information using the short forms helps in reducing the confusion. The candidates may also, where possible try to correlate the given relationships with their own kith and kin. This works wonderfully in understanding the relationship.

It is observed that the names given in the questions are sometimes deceptive as to the gender. Candidates are advised to follow the instructions given in the question ignoring the name of the gender as at times it is deceptive. e.g., Kamal, Milan, Preetam, Kiran, Jasbir, Jasprit and etc.

Quite often descriptions of superfluous (unnecessary/ redundant) are given. It is thus better to first identify relationship between which two persons is exactly required to be found out in the question. And then proceed to track the relationship based on the descriptions connected to them. Again properly understanding the relationship between which two persons is exactly required to be found out in the question is important as the relationship between A and B would be different

than between B and A. For example, if Ravi and Mala are brother and sister, Ravi is related to Mala as brother, whereas Mala is related to Ravi as sister.

Also remember that terms like „only son“ only means that the person do not have another son, but it does not mean that the person doesn't have daughter/s. However, when it is said a person does not have any brothers and sisters, it can be safely concluded that he is the only child of his father/mother.

Quite often a candidate depending upon whether he is male or female presumes that a person whose sex is not explicitly referred to in the relationships is of the same sex as that of his/her. Such bias about the sex of the persons referred to the relationships should be avoided and the candidate should go strictly by the description given in the question.

Often the relationships are described in a roundabout way. Some examples of such descriptions and their actual/direct meaning is given below:

- a) Only son of my grandfather – My Father
- b) Only son of my grandmother – My Father
- c) Only daughter of my grandfather – My Mother
- d) Only daughter of my grandmother – My Mother
- e) Sister of my mother – My Aunt
- f) Son/Daughter of my husband – My son / daughter
- g) Son/Daughter of my wife – My son / daughter
- h) Only daughter of my grandfather's only son – My Sister
- i) Grandmother of my father's only son – My Grandmother
- j) Father of my daughter's father – My Father k) Father of my son's father – My Father
- l) My son's sister – My daughter
- m) Daughter-in-law of grandmother of my father's only son – My Mother
- n) A is the father of B but B is not the son of A – B is daughter of A

After the answer is found, it is good practice to quickly check back the answers with the relevant information given in the question.



Example: - Pinky, who is Victor's daughter, say to Lucy, "Your Mother Rosy is the younger sister of my Father, who is the third child of Joseph." How is Joseph related to Lucy?

- a) Father - In - Law
- b) Father
- c) Maternal Uncle
- d) Grand Father

Solution

(Option D)



Example: - Mohan is the Son of Arun's Father's sister. Prakash is the son of Reva, who is the mother of Vikas and Grandmother of Arun. Pranab is the father of Neela and the grandfather of Mohan. Reva is the wife of Pranab. How is the wife of Vikas related to the Neela?

- a) Sister

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- b) Sister - In - Law
- c) Niece
- d) None of The Above

Solution

(Option B)



Example: T, S and R are three brothers. T's son Q is married to K and they have one child Rahul blessed to them. M the son of S is married to H and this couple is blessed with a daughter Madhvi. R has a daughter N who is married to P. This couple has one daughter Karuna born to them. How is Madhvi related to S?

- a) Daughter
- b) Niece
- c) Grand Daughter
- d) None of The Above

Solution

(Option C)



Example: - Arti and Saurabh are the children of Mr and Mrs Shah. Ritu and Shakti are the children of Mr and Mrs. Mehra. Saurabh and Ritu are married to each other and two daughter Mukti and Shruti are born to them. Shakti is married to Rina and two children Subhash and Reshma are born to them. How is Arti related to Shruti?

- a) Mother
- b) Mother - in - Law
- c) Sister
- d) Aunt

Solution

(Option D)

Summary

The key concepts learned from this unit are: -

- We have learnt how find Family or Blood Relationship.
- We have learnt how find Relationships Involving the Term „-in-law“ General
- We have learnt how find Relationships Involving Father-in-law, Mother-in-law, Son-in law and Daughter-in-law
- We have learnt how find Relationships Involving Brother-in-law and Sister-in-law

- We have learnt how find Relationships Half Sibling and Step Relations

Keywords

- Blood Relationship
- Relationships bases Puzzles
- Coded Relations

Self Assessment

1. Anil, introducing a girl in a party, said, she is the wife of the 12, grandson of my mother. How is Anil related to the girl?

- A. Father
- B. Grandfather
- C. Husband
- D. Father-in-law

2. A man said to a woman, —Your mother's husband's sister is my aunt. | How is the woman related to the man?

- A. Granddaughter
- B. Daughter
- C. Sister
- D. Aunt

3. Introducing Rajesh, Neha said, —His brother's father is the only son of my grandfather|. How Neha is related to Rajesh?

- A. Sister
- B. Daughter
- C. Mother
- D. Niece

4. Vinod is the brother of Bhaskar. Manohar is the sister of Vinod. Biswal is the brother of Preetam and Preetam is the daughter of Bhaskar. Who is the uncle of Biswal?

- A. Bhaskar
- B. Manohar
- C. Vinod
- D. Insufficient data

5. A man said to a woman, —Your brother's only sister is my mother. | What is the relation of the woman with the maternal grandmother of that man?

- A. Mother
- B. Sister
- C. Niece

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D. Daughter

6. Pointing to a photograph, a man said, —I have no brother or sister but that man's father is my father's son. | Whose photograph, was it?

A. His own

B. His son's

C. His father's

D. His nephew's

7. Pointing to a photograph, a lady tells Pramod, —I am the only daughter of this lady and her son is your maternal uncle, | How is the speaker related to Pramod's father?

A. Sister-in-law

B. Wife

C. Neither (a) nor (b)

D. Aunt

8. A is the brother of B. A is the brother of C. To find what is the relation between B and C. What minimum information from the following is necessary?

(i) Gender of C (ii) Gender of B

A. Only (i)

B. Only(ii)

C. Either (i) or (ii)

D. both (i) and (ii)

9. Looking at a portrait of a man, Harsh said, "His mother is the wife of my father's son. Brothers and sisters, I have none. "At whose portrait was Harsh looking?

A. His son

B. His cousin

C. His uncle

D. His nephew

10. A, B, C, D, E, F and G are members of a family consisting of 4 adults and 3 children, two of whom, F and G are girls. A and D are brothers and A is a doctor. E is an engineer married to one of the brothers and has two children. B is married to D and G is their child. Who is C?

A. G's brother

B. F's father

C. E's father

D. A's son

11. In a family of 5, P is the father of R. S is Q's son. S has R as sister. Therefore, if U has P as brother, then the relationship between Q and U is as follows.

A. Q is U's daughter

- B. U is Q's wife
 C. Q is the sister-in-law of U
 D. Q is U's brother-in-law

12. A, B and C are sisters. D is the brother of E and E is the daughter of B. How is A related to D?

- A. Sister
 B. Cousin
 C. Niece
 D. Aunt

13. F is the brother of A. C is the daughter of A. K is the sister of F. G is the brother of C. Who is the uncle of G?

- A. A
 B. C
 C. F
 D. K

14. P is the brother of Q and R. S is the R's mother. T is P's father. Which of the following statements cannot be definitely true?

- A. T is Q's father
 B. S is P's mother
 C. T is S's husband
 D. S is T's son

15. A party consisted of a man, his wife, his three sons and their wives and three children in each son's family. How many were there in the party?

- A. 24
 B. 22
 C. 13
 D. 17

Answers for Self Assessment

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. D | 2. C | 3. A | 4. C | 5. D |
| 6. B | 7. B | 8. D | 9. A | 10. D |
| 11. C | 12. D | 13. C | 14. D | 15. D |

Review Questions

1. Pointing to Ajay, Radha said, "His father is the only son of my grandfather". How is Radha Related to Ajay?

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2. Lalita said to Tina, "You are the daughter-in-law of the grandmother of my father's only son."
3. Pointing to a photograph, Amar said, "I have no brother or sister but that man's father is my father's son." Whose photograph, was it? How is Lalita related to Tina?
4. Looking at the portrait of a man, Ashok said, „His mother is the wife of my father's son. Brothers and sisters, I have none". At whose portrait was Ashok looking?
5. Ahmad said to Saira, „Your only brother's son is my wife's brother". How is Saira related to the Ahmad's wife?
6. Pointing to a gentleman, Abdul said, "His only brother is the father of my daughter's father". How is the gentleman related to Abdul?"
7. Pointing to a man in a photograph, Malati tells, "His brother's father is the only son of my grandfather." How is Malati related to the man in the photograph?
8. Pointing to Dharmendra, Hema said, "He is the son of my father's only son." How is Dharmendra's mother related to Hema?
9. Mr. „Ashok" meets Mr. „Babu". „Babu" is the father of a son „Dharmendra" and a daughter „Chandrika". „Shalini" is the mother of „Ashok" „Dharmendra" is married has one son. „Shalini" is the daughter-in-law of „Babu". How is „Ashok" related to „Babu".
10. Pointing to a photograph, a lady tells Bhushan, "I am the only daughter of this lady and her son is your maternal uncle." How is the speaker related to Bhushan's father?

**Further Reading**

1. A Modern Approach to Verbal & Non-Verbal Reasoning by Dr. R.S. Aggarwal, SChand Publishing
2. Analytical Reasoning by M.K. Pandey, Banking Service Chronicle

**Web Links**

1. <https://www.examveda.com/mcq-question-on-competitive-reasoning/>
2. <https://www.hitbullseye.com/Reasoning>

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